A

COMPLETE KEY

TO

GUMMERE'S SURVEYING;

IN WHICH

THE OPERATIONS OF ALL THE EXAMPLES NOT SOLVED IN THAT WORK ARE EXHIBITED AT LARGE.

PRINCIPALLY DESIGNED

TO FACILITATE THE LABOUR OF TEACHERS,

AND TO ASSIST THOSE

WHO HAVE NOT THE OPPORTUNITY OF THEIR INSTRUCTION.

By SAMUEL ALSOP.

ADAPTED TO THE REVISED EDITION OF THE SURVEYING

By ISAAC SHARPLESS,
Author of "A Text-book of Geometry and Trigonometry."

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KEY

TO

GUMMERE'S SURVEYING.

PLANE TRIGONOMETRY.

CASE 1.

EXAMPLE 3. (Pl. 1, fig. 1.)

Angle C=180°—A—B=46° 15′.

As sin. A 79° 23′ - - - - - - - -	Ar. Co.	0.007499
Is to sin. B 54° 22′ - - - - - - - - - - -		9.909963
So is BC 125 - - - - - - - - - - - -		2.096910
To AC 103.4 - - - - - - - - - - - -		2.014372

Again,

As sin. A - - - - - - - - - - -	Ar. Co.	0.007499
Is to sin. C 46° 15′ - - - - - - - - - - -		9.858756
So is BC - - - - - - - - - - - - -		2.096910
To AB 91.87 - - - - - - - - - - - -		1.963165

EXAMPLE 4. (Pl. 1, fig. 2.)

Angle C=90°—A=33° 12′.

As sin. C 33° 12′ - - - - - - - -	Ar. Co.	0.261566
Is to sin. B 90 - - - - - - - - - - - -		10.000000
So is AB 53.66 - - - - - - - - - - -		1.729651
To AC 98 - - - - - - - - - - - - -		1.991217

As sin. C - - - - - - - - - - -	Ar. Co.	0.261566
Is to sin. A 56° 48′ - - - - - - - - - - -		9.922603
So is AB - - - - - - - - - - - - -		1.729651
To BC 82 - - - - - - - - - - - - -		1.913820

Example 5. (Pl. 1, fig 2.)

Angle C=90°—A=50° 50′.

As sin. A 39° 10′	Ar. Co.	0.199573
Is to sin. B 90°		10.000000
So is BC 407.37		2.609989
To AC 645		2.809562
As sin. A	Ar. Co.	0.199573
Is to sin. C 50° 50′		9.889477
So is BC		2.609989
To AB 500.1		2.699039

CASE 2.

Example 3. (Pl. 1, fig. 1.)

As AC 306	Ar. Co.	7.514278
Is to AB 274		2.437751
So is sin. B 78° 13′		9.990750
To sin. C 61 14		9.942779
139 27		
180		
A 40 33		
As sin. B 78° 3′	Ar. Co.	0.009515
Is to sin. A 40° 33′		9.812988
So is AC 306		2.485721
To BC 203.4		2.308224

Example 4. (Pl. 1, fig. 2.)

As AC 272	Ar. Co.	7.565431
Is to AB 232		2.365488
So is sin. B 90°		10.000000
To sin. C 58° 32′		9.930919
A 31° 28′		

As sin. B	Ar. Co.	0.000000
Is to sin. A 31° 28′		9.717673
So is AC		2.434569
To BC 142		2.152242

EXAMPLE 5. (Pl. 1, fig. 2.)

As AC 150	Ar. Co.	7.823909
Is to BC 69		1.838849
So is sin. B 90°		10.000000
To sin. A 27° 23′		9.662758
C 62° 37′		

As sin. B	Ar. Co.	0.000000
Is to sin. C 62° 37′		9.948388
So is AC		2.176091
To AB 133.2		2.124479

CASE 3.

EXAMPLE 2. (Pl. 1, fig. 3.)

$$\frac{A+C}{2}=\frac{180°-B}{2}=39°\ 15'$$

As AB+BC 185	Ar. Co.	7.732828
Is to AB—BC 33		1.518514
So is tang. $\frac{C+A}{2}$ 39° 15′		9.912240
To tang. $\frac{C-A}{2}$ 8° 18′		9.163582
C 47° 33′		

As sin. A 30° 57′	Ar. Co.	0.288792
Is to sin. B 101° 30′		9.991193
So is BC 76		1.880814
To AC 144.8		2.160799

Example 3. (Pl. 1, fig. 2.)

$$\frac{C+A}{2}=\frac{180°-B}{2}=45°$$

As AB+BC 1677 - - - - - - - -	Ar. Co.	6.775467
Is to AB—BC 103 - - - - - - - - - - -		2.012837
So is tang. $\frac{C+A}{2}$ - - 45° - - - - - - - -		10.000000
To tang. $\frac{C-A}{2}$ - - 3° 31′ - - - - - -		8.788304
C - - 48° 31′		
As sin. A 41° 29′ - - - - - - - -	Ar. Co.	0.178878
Is to sin. B 90 - - - - - - - - - - - -		10.000000
So is BC 787 - - - - - - - - - - - -		2.895975
To AC 1188 - - - - - - - - - - - -		3.074853

CASE 4.

RULE 2.

EXAMPLE 2. (Pl. 1, fig. 4.)

AC	47		
AB	64	Ar. Co.	8.193820
BC	34	Ar. Co.	8.468521
	2)145		
Half sum	72.5	- - - - -	1.860338
Difference	25.5	- - - - -	1.406540
			2)19.929219
Cos. ½ B	22° 49′	- - -	9.964609
B	45° 38′		

EXAMPLE 3. (Pl. 1, fig. 4.)

AB	108		
BC	54	Ar. Co.	8.267606
AC	88	Ar. Co.	8.055517
	2)250		
Half sum	125	- - - - - -	2.096910
Difference	17	- - - - - -	1.230449
			2)19.650482
Cos. ½ C	48° 2′	- - -	9.825241
C	96° 4′		

RIGHT ANGLED TRIANGLES.

First Method.

EXAMPLE 2. (Pl. 1, fig. 5.)

Making AC radius, CB is sine of A, and AB is cos. A; hence,

As radius	Ar. Co.	0.000000
Is to sin. A 27° 46′		9.668267
So is AC 36.57		1.563125
To BC 17.04		1.231392

And,

As radius	Ar. Co.	0.000000
Is to cos. A		9.946871
So is AC		1.563125
To AB 32.36		1.509996

Example 3. (Pl. 1, fig. 5.)

Making AC radius, we have CB the sine, and AB the cosine of A; hence,

As sin. A 42° 9′	Ar. Co.	0.173230
Is to radius		10.000000
So is BC 193.6		2.286905
To AC 288.5		2.460135

And,

As sin. A	Ar. Co.	0.173230
Is to cos. A		9.870047
So is BC		2.286905
To AB 213.9		2.330182

Example 4. (Pl. 1, fig. 6.)

Making the base AB radius, we have BC the tangent of A; making AC the radius, we have AB the cosine of A; hence,

As AB 46.72	Ar. Co.	8.330497
Is to BC 57.9		1.762679
So is rad.		10.000000
To tang. A 51° 6′		10.093176

And,

As cos. A	Ar. Co.	0.202066
Is to rad.		10.000000
So is AB		1.669503
To AC 74.4		1.871569

Second Method.—By Logarithms.

EXAMPLE 3.

Hypothenuse	403		
Base - - -	321		
Sum - - - -	724	- - - log.	2.859739
Difference - -	82	- - - - "	1.913814
			2)4.773553
Perpendicular	243.65	- - - -	2.386776

EXAMPLE 4.

Perpendicular	27.2	- - - log.	1.434569	
			2.869138	
Base - - -	31.04	- - - -	1.491922	1.491922
	23.835	- - - -	1.377216	
	54.875	- - -	- - -	1.739374
				2)3.231296
Hypothenuse	41.27	- - - -	- - -	1.615648

APPLICATION OF PLANE TRIGONOMETRY TO THE MENSURATION OF DISTANCES AND HEIGHTS.

EXAMPLE 1. (*See* fig. 54, *Surveying*.)

Angle C=180°—A—B=56° 23′.

To find AC:

As sin. C 56° 23′ - - - - - - - -	Ar. Co.	0.079480
Is to sin. B 49° 23′ - - - - - - - - - -		9.880289
So is AB 500 yards - - - - - - - - - -		2.698970
To AC 455.8 - - - - - - - - - - - -		2.658739

To find BC:

As sin. C	Ar. Co.	0.079480
Is to sin. A 74° 14′		9.983345
So is AB		2.698970
To BC 577.8		2.761795

Example 2. (Fig. 55, *Surveying*.)

As BC+AC 1575	Ar. Co.	6.802719
Is to BC—AC 105		2.021189
So is tang. $\frac{A+B}{2}$ 62° 10′		10.277379
To tang. $\frac{A-B}{2}$ 7° 12′		9.101287
B 54° 58′		

As sin. B	Ar. Co.	0.086813
Is to sin. C 55° 40′		9.916859
So is AC 735		2.866287
To AB 741.2		2.869959

Example 3. (Fig. 56, *Surveying*.)

Angle CAD=180—ADC—ACD=31° 10′

To find AC:

As sin. CAD 31° 10′	Ar. Co.	0.286065
Is to sin. ADC 53° 30′		9.905179
So is CD 300		2.477121
To AC 465.98		2.668365

Angle CBD=180—BCD—BDC=22° 55′

To find CB:

As sin. CBD 22° 55′	Ar. Co.	0.409613
Is to sin. CDB 98° 45′		9.994916
So is CD		2.477121
To CB 761.47		2.881650

To find AB:

As BC+AC 1227.45		Ar. Co.	6.910995
Is to BC—AC 295.49			2.470542
So is tang. $\frac{CAB+CBA}{2}$	71° 30′		10.475480
To tang. $\frac{CAB—CBA}{2}$	35° 44′		9.857017
CBA	35° 46		

As sin CBA		Ar. Co.	0.233226
Is to sin. BCA 37°			9.779463
So is CA 465.98			2.668367
To AB 479.8			2.681056

EXAMPLE 4. (Fig. 57, *Surveying.*)

To find C:

AB	3		
AC	2	Ar. Co.	9.698970
BC	1.8	Ar. Co.	9.744727
	2)6.8		
Half sum	3.4		0.531479
Difference	.4		—1.602060
			2)19.577236
Cos. ½ C	52° 4′		9.788618
C	104° 8′		

To find BD:

As sin. D 17° 47′		Ar. Co.	0.515105
Is to sin. C 104° 8′			9.986651
So is BC 1.8			0.255273
To BD 5.715			0.757029

To find CD:

As sin. D	Ar. Co	0.515105
Is to sin. DBC 58° 5′		9.928815
So is BC		0.255273
To CD 5.003		0.699193

Example 5. (Fig. 58, *Surveying*.)

To find BAC:

BC	7.2		
AB	12	Ar. Co.	8.920819
AC	8	Ar. Co.	9.096910
	27.2		
Half sum	13.6		1.133539
Difference	6.4		0.806180
			2)19.957448
Cos. ½ BAC	17° 47′½		9.978724
BAC	35° 35		

To find AE:

As sin. AEB 136°	Ar. Co.	0.158229
Is to sin. EBA 19°		9.512642
So is AB 12		1.079181
To AE 5.624		0.750052

To find ACE:

As AC+AE 13.624		Ar. Co.	8.865696
Is to AC—AE 2.376			0.375846
So is tang. $\frac{AEC+ACE}{2}$	84° 42′½		11.033291
To tang. $\frac{AEC—ACE}{2}$	62 1½		10.274833
AEC	146 44		
ACE	22° 41′		

To find AD:

As sin. ADC 19°	Ar. Co.	0.487358
Is to sin. ACD 22° 41′		9.586179
So is AC 8		0.903090
To AD 9.476		0.976627

To find CD:

As sin. ADC	Ar. Co.	0.487358
Is to sin. DAC 138° 19′		9.822830
So is AC		0.903090
To DC 16.34		1.213278

To find BD:

As sin. ADB 44°	Ar. Co.	0.158229
Is to sin. BAD 102° 44′		9.989186
So is AB 12		1.079181
To DB 16.85		1.226596

EXAMPLE 6. (Fig. 59, *Surveying*.)

To find ABC:

AC	46		
AB	50	Ar. Co.	8.301030
BC	40	Ar. Co.	8.397940
	2)136		
Half sum	68		1.832509
Difference	22		1.342423
			2)19.873902
Cos. ½ ABC	30° 8′		9.936951
ABC	60° 16′		

To find CD and CE:

As sin. ADC 60° 16′	Ar. Co.	0.061309
Is to radius		10.000000
So is AC 46		1.662758
To CD 52.98		1.724067

$CE = \frac{1}{2} CD = 26.49$

Also CAE=90°—ADC=90°—ABC=29° 44′.

EXAMPLE 7. (Fig. 61, *Surveying.*)

Making DE radius, EC is tangent of D; hence,

As radius	Ar. Co.	0.000000
Is to tang. D 47° 30′		10.037948
So is DE 100		2.000000
To EC 109.13		2.037948
EB 5		
BC 114.13		

EXAMPLE 8. (Fig. 62, *Surveying.*)

To find DC:

As sin. ACD 25°	Ar. Co.	0.374052
Is to sin. CAD 26° 30′		9.649527
So is AD 75 ft.		1.875061
To DC 79.18 ft.		1.898640

To find BC:

As radius	Ar. Co.	0.000000
Is to sin. CDB 51° 30′		9.893544
So is CD		1.898640
To CB 61.97 ft.		1.792184

EXAMPLE 9. (Fig. 63, *Surveying.*)

To find DC:

As sin. ACD 23° 50′	Ar. Co.	0.393535
Is to sin. CAD 44°		9.841771
So is AD 134		2.127105
To DC 230.4		2.362411

To find CE:

As sin. CED 141°	Ar. Co.	0.201128
Is to sin. CDE 16° 50′		9.461782
So is CD		2.362411
To CE 106		2.025321

To find EB:

As radius	Ar. Co.	0.000000
Is to sin. CDB 67° 50′		9.966653
So is CD		2.362411
To CB 213.3		2.329064

CE 106

BE 107.3

Example 10. (Fig. 64, *Surveying*.)

To find BD:

As sin. CDB 17° 15′	Ar. Co.	0.527914
Is to sin. BCD 23° 45′		9.605032
So is CB 60		1.778151
To BD 81.49		1.911097

To find AD:

As BD+BA 121.49	Ar. Co.	7.915460
Is to BD—BA 41.49		1.617943
So is tang. $\frac{BAD+BDA}{2}$ 69° 30′		10.427262
To tang. $\frac{BAD-BDA}{2}$ 42° 25′		9.960665

BDA 27° 5′

As sin. ADB 27° 5′	Ar. Co.	0.341716
Is to sin. ABD 41°		9.816943
So is AB 40		1.602060
To AD 57.64		1.760719

EXAMPLE 11. (Fig. 65, *Surveying.*)

To find AD:

As sin. CAD 27°	Ar. Co.	0.342953
Is to sin. ACD 138°		9.825511
So is CD 132		2.120574
To AD		2.289038

To find AB:

As sin. ABD 109°	Ar. Co.	0.024330
Is to sin. ADB 8°		9.143555
So is AD		2.289038
To AB 28.64		1.456923

PRACTICAL QUESTIONS.

EXAMPLE 1. (Pl. 1, fig. 2.)

Making AB radius, BC is tangent of A.

As radius	Ar. Co.	0.000000
Is to tang. A 52° 30′		10.115020
So is AB 85		1.929419
To BC 110.8		2.044439

EXAMPLE 2. (Pl. 1, fig. 2.)

Make AB radius, then will BC be the tangent of A; making AC radius, AB will be the cosine of A; hence,

As radius	Ar. Co.	0.000000
Is to tang. A 61° 45′		10.269767
So is AB 73		1.863323
To BC 135.9		2.133090

And,

As cosine A	Ar. Co.	0.324845
Is to radius		10.000000
So is AB		1.863323
To AC 154.2		2.188168

EXAMPLE 3. (Pl. 1, fig. 7.)

To find BD. We have in the triangle ABD, the angles and side AB. Hence,

As sin. ADB 31°	Ar. Co.	0.288161
Is to sin. BAD 100°		9.993351
So is AB 339		2.530200
To BD 648.2		2.811712

Again, in ABC we have the angles, and side AB, to find BC. Thus,

As sin. ACB 22° 30′ - - - - - -	Ar. Co.	0.417160
Is to sin. BAC 36° 30′ - - - - - - - - - -		9.774388
So is AB 339 - - - - - - - - - - - - -		2.530200
To BC 526.9 - - - - - - - - - - - - -		2.721748

In DBC we have the sides DB and BC, and included angle DBC= 72°. To find the side DC. Thus

$$BCD+BDC=180°-72°=108°.$$

Then,

As BD+BC 1175.1 - - - - - - -	Ar. Co.	6.929925
Is to BD—BC 121.3 - - - - - - - - - - -		2.083861
So is tang. $\frac{BCD+BDC}{2}$ - 54° - - - - - -		10.138739
To tang. $\frac{BCD-BDC}{2}$ - 8° 5′ - - - - -		9.152525
BDC 45° 55′		

And,

As sin. BDC 45° 55′ - - - - - - -	Ar. Co.	0.143677
Is to sin. DBC 72° - - - - - - - - - - -		9.978206
So is BC - - - - - - - - - - - - - -		2.721748
To CD 697.64 - - - - - - - - - - - - -		2.843631

This example might have been solved by finding AD=496.76, AC=759.33; whence the angle ADC would be found to be 76° 55′, and CD=697.64, as before.

Example 4. (Pl. 1, fig. 8.)

Construction.

With the given distances construct the triangle ABC. Make ACE and CAE respectively equal to 13° 30′ and 29° 50′. About the triangle AEC describe the circle ACD. Join EB, and produce it to meet the circumference in D, which will be the situation of the observer.

Since the angles ADE and ACE are subtended by the same arc, we have ADE=ACE=13° 30′. Also CDE=CAE=29° 50′.

Calculation.

In the triangle ABC, we have the three sides to find the angle BAC. Thus,

BC	262		
AC	404	Ar. Co.	7.393619
AB	213	Ar. Co.	7.671620
	2)879		
Half sum	439.5	- - - - - -	2.642959
Difference	177.5	- - - - - -	2.249198
			2)19.957396
Cos. ½ BAC	17° 48′	- - -	9.978698
BAC	35° 36′		

In the triangle ACE we have the angles and side AC, to find AE.

As sin. AEC 136° 40′ - - - - - - Ar. Co.	0.163523
Is to sin. ACE 13° 30′ - - - - - - - - - -	9.368185
So is AC 404 - - - - - - - - - - - - -	2.606381
To AE 137.43 - - - - - - - - - - - - -	2.138089

In the triangle ABE we have the sides AB and AE, and the included angle BAE=BAC+CAE=65° 26′. To find ABE, thus:

As AB+AE 350.43	- - - - - -	Ar. Co.	7.455399
Is to AB—AE 75.57	- - - - - - - - - -		1.878349
So is tang. $\frac{AEB+ABE}{2}$	57° 17′	- - - -	10.192195
To tang. $\frac{AEB-ABE}{2}$	18° 33′	- - - -	9.525943
	ABE=38° 44′		

In ABD we have ABD=180°—38° 44′=141° 16′, ADB=13° 30′, and BAD=38° 44′—13° 30′=25° 14′. To find AD and DB:

As sin. ADB 13° 30′ - - - - - - Ar. Co.	0.631815
Is to sin. BAD 25° 14′ - - - - - - - - - -	9.629721
So is AB 213 - - - - - - - - - - - - -	2.328380
To BD 389 - - - - - - - - - - - - - -	2.589916

And,

As sin. ADB 13° 30′	Ar. Co.	0.631815
Is to sin. ABD 141° 16′		9.796364
So is AB 213		2.328380
To AD 570.9		2.756559

Finally, in ADC we have the angle ADC=43° 20′, CAD=BAC+BAD=60° 50′ and the side AC; to find CD. Thus,

As sin. ADC 43° 20′	Ar. Co.	0.163523
Is to sin. CAD 60° 50′		9.941117
So is AC 404		2.606381
To CD 514.1		2.711021

This might have been solved by finding ACB = 28° 14′, CE = 292.87, whence CBE would have been found to be = 77° 26′, BD = 388.9, DC = 514, and AD = 570.8.

EXAMPLE 5. (Pl. 1, Fig. 9.)

Here $AD = \sqrt{BD^2 - AB^2} = \sqrt{1296} = 36$.
And $AC = AD + DC = 75$ Ans.

Or, Trigonometrically;

As BD 39	Ar. Co.	8.408935
Is to BA 15		1.176091
So is radius		10.000000
To cos. B 67° 23′		9.585026

And,

As radius	Ar. Co.	0.000000
Is to sin. B 67° 23′		9.965248
So is BD 39		1.591065
To AD 36		1.556313
AC 75		

EXAMPLE 6. (Pl. 1, fig. 10.)

The angle ACB = DBC — DAC = 25°.

Then,

As sin ACB 25°	Ar. Co.	0.374052
Is to sin. BAC 26° 30′		9.649527
So is AB 75		1.875061
To BC 79.18		1.898640

To find CD, and BD:

As radius	Ar. Co.	0.000000
Is to sin. B 51° 30′		9.893544
So is CB		1.898640
To CD 61.97		1.792184

And,

As radius	Ar. Co.	0.000000
Is to cos. B		9.794150
So is CB		1.898640
To BD 49.29		1.692790

EXAMPLE 7. (Pl. 1, fig. 11.)

Here ACB = CAD = 35° and BAC = 55°

Hence,

As rad.	Ar. Co.	0.000000
Is to tan. BAC 55°		10.154773
So is AB 143.		2.155336
To BC 204.2		2.310109

EXAMPLE 8. (Pl. 1, fig. 12.)

Construction.

Make AB=76, the distance from the lower column to the statue's base. Erect the perpendiculars AD and BF, making the former= 50. With D as a centre and distance 86, cross BF in F, which will be the head of the statue.

Make AI = 64, draw IE parallel to AC, with F as a centre and distance 97, cross IE in E, then EC perpendicular to AC, will be the higher column.

Calculation.

To find FDG and side DG:

As DF 86	Ar. Co.	8.065502
Is to FG 76		1.880814
So is radius		10.000000
To sin. FDG 62° 5½′		9.946316
As radius	Ar. Co.	0.000000
Is to cos. FDG 62° 5½′		9.670300
So is FD 86		1.934498
To DG 40.25		1.604798

To find EFH and FH, we have FE = 97 and EH = GI = GD + DI = 54.25. Hence,

As EF 97	Ar. Co.	8.013228
Is to EH 54.25		1.734400
So is radius		10.000000
To sin. EFH 34°		9.747628

And,

As radius	Ar. Co.	0.000000
Is to cos. F 34°		9.918574
So is EF 97		1.986772
To FH 80.42		1.905346

To find ED, we have EI = HF + FG = 156.42 and DI = 14. Hence,

As IE 156.42	Ar. Co.	7.805707
Is to ID 14		1.146128
So is radius		10.000000
To tan. IED 5° 7′		8.951835

As cos. E 5° 7′ - - - - - - - - - -	Ar. Co.	0.001734
Is to rad. - - - - - - - - - - - - - -		10.000000
So is IE 156.42 - - - - - - - - - - - -		2.194293
To ED 157.04 - - - - - - - - - - - - -		2.196027

Otherwise.

$GD = \sqrt{FD^2 - FG^2} = \sqrt{1620} = 40.25.$

$GI = GD + DI = 54.25.$

$FH = \sqrt{FE^2 - EH^2} = \sqrt{6465.9375} = 80.41.$

$IE = FH + FG = 156.41.$

$ID = \sqrt{IE^2 + ID^2} = \sqrt{24660.0881} = 157.03.$

SURVEYING.

CHAPTER I.

DIMENSIONS OF A SURVEY.

PROBLEM 8.

EXAMPLE 2.

Angle B = 34°+35° = 69°.

EXAMPLE 3.

Here the first bearing must be reversed, since it is towards the station C. It becomes N. 35° W. Hence C=180°—(35°+87°)=58°.

EXAMPLE 4.

D = 180° — (87° — 58°) = 151°.

PROBLEM 9.

EXAMPLE 2.

1st side	S.	40½°	E.	3d	N. 29¼° E.
	N	54	E.		N 54 E.
		94½			N. 24¾ W.
		180			
		N. 85½	E.		

4th	N.	28¾°	E.	5th	N. 57° W.
	N.	54	E.		N. 54 E.
	N.	25¼	W.		111
					180
					S. 69 W.

6th	S.	47°	W.
	N.	54	E.
	S.	7	E.

EXAMPLE 3.

1st	S.	45½°	W.	2d	N.	50°	W.
	S.	20½	W.		S.	20½	W.
	S.	25	W.		N.	70½	W.
3d	N.	0°	W.	4th	N.	85°	E.
	S.	20½	W.		S.	20½	W.
	N.	20½	W.		N.	64½	E.
5th	S.	47°	E.	7th	N.	51¼°	W.
	S.	20½	W		S.	20½	W.
	S.	67½	E.		N.	71¾	W.

PROBLEM 10.

EXAMPLE 1.

As sin. bearing 32° 30′ - - - - - - -	Ar. Co.	0.269784
Is to radius - - - - - - - - - - - - -		10.000000
So is departure 10.96 - - - - - - - - - -		1.039811
To distance 20.40 - - - - - - - - - - -		1.309595

And,

As radius - - - - - - - - - - - -	Ar. Co.	0.000000
Is to cotangent bearing - - - - - - - - -		10.195813
So is departure - - - - - - - - - - - -		1.039811
To difference of latitude 17.20 - - - - - - -		1.235624

EXAMPLE 2.

As distance 44	Ar. Co.	8.356547
Is to difference of latitude 34.43		1.536937
So is radius		10.000000
To cosine of bearing 38° 31′		9.893484

And,

As rad.	Ar. Co.	0.000000
Is to tang. bearing 38° 31′		9.900864
So is diff. lat. 34.43		1.536937
To departure 27.40		1.437801

EXAMPLE 3.

As cosine of bearing 32° 30′	Ar. Co.	0.073971
Is to radius		10.000000
So is diff. of lat. 17.21		1.235781
To distance 20.41		1.309752

And,

As radius	Ar. Co.	0.000000
Is to tang. bearing 32° 30′		9.804187
So is diff. latitude 17.21		1.235781
To departure 10.96		1.039968

EXAMPLE 4.

As diff. of lat. 27.92 N.	Ar. Co.	8.554085
Is to departure 5.32 E.		0.725912
So is radius		10.000000
To tang. bear. 10° 47′		9.279997

And,

As cosine bearing 10° 47′	Ar. Co.	0.007737
Is to radius		10.000000
So is diff. of lat.		1.445915
To dist. 28.42		1.453652

EXAMPLE 5.

As distance 35.35 - - - - - - -	Ar. Co.	8.451611
Is to departure 15.08 - - - - - - - - -		1.178401
So is radius - - - - - - - - - - - -		10.000000
To sin. bearing 25° 15′ - - - - - - - -		9.630012

And,

As radius - - - - - - - - - - -	Ar. Co.	0.000000
Is to cos. bearing 25° 15′ - - - - - - -		9.956387
So is distance - - - - - - - - - - - -		1.548389
To diff. of lat. 31.97 - - - - - - - -		1.504776

PROBLEM 12.

Sta.	Courses.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. E.	N.	S.	E.	W.
1	N. 75 E.	13.70	3.54		13.24		2	2	3.56		13.26	
2	N. 20½ E.	10.30	9.65		3.61		1	1	9.66		3.62	
3	East.	16.20			16.20		2	2	.02		16.22	
4	S. 33½ W.	35.30		29.44		19.49	5	5		29.39		19.44
5	S. 76 W.	16.00		3.87		15.52	2	2		3.85		15.50
6	North.	9.00	9.00				1	1	9.01		.01	
7	S. 84 W.	11.60		1.21		11.54	2	2		1.19		11.52
8	N. 53¼ W.	11.60	6.94			9.29	2	2	6.96			9.27
9	N. 36¾ E.	19.36	15.51		11.59		3	2	15.54		11.61	
10	N. 22½ E.	14.00	12.93		5.36		2	2	12.95		5.38	
11	S. 76¾ E.	12.00		2.75	11.68		2	2		2.73	11.70	
12	S. 15 W.	10.85		10.48		2.81	2	1		10.46		2.80
13	S. 18 W.	10.62		10.10		3.28	2	1		10.08		3.27
			57.57	57.85 57.57	61.68	61.93 61.68						

Error South .28 .25 Error West.

CHAPTER II.

SUPPLYING OMISSIONS.

PROBLEM I.

EXAMPLE 2.

Sta.	Courses.	Dist.	N.	S.	E.	W.
1	N. 15¾° W.	9.40	9.05			2.55
2	N. 63¾ E.	10,43	4.61		9.36	
3	S. 49 E.	8.12		5.33	6.13	
4	S. 13½ E.	8.45		8.22	1.98	
5	S. 16¾ E.	6.44		6.17	1.86	
6				(6.11)		(10.64)
7	N. 60 W.	9.72	4.86			8.41
8	N. 17¼ W.	7.65	7.31		2.27	
			25.83	25.83	21.60	21.60

Then,

As diff. lat. 6.11 S. - - - - - - - - Ar. Co. 9.213959
Is to depart. 10.64 W. - - - - - - - - - - 1.026942
So is radius - - - - - - - - - - - - - - 10.000000

To tang. bearing S. 60° 8′ W. - - - - - - - - 10.240901

And, As cosine bearing 60° 8′ - - - - - - Ar. Co. 0.302785
Is to radius - - - - - - - - - - - - - - - 10.000000
So is diff. lat. - - - - - - - - - - - - - 0.786041

To distance 12.27 - - - - - - - - - - - - - 1.088826

EXAMPLE 3.

Sta.	Courses.	Dist.	N.	S.	E.	W.
1	S. 52° W.	10.70		6.59		8.43
2	S. 7½ W.	13.92		13.80		1.82
3	S. 34¼ E.	9.00		7.44	5.07	
4			(27.83)		(5.18)	
				27.83	10.25	10.25

Then,

As diff. lat. 27.83	Ar. Co.	8.555487
Is to departure 5.18		0.714330
So is radius		0.000000
To tang. bearing N. 10° 33′ E.		9.269817

And,

As cosine bearing 10° 33′	Ar. Co.	0.007404
Is to radius		10.000000
So is diff. lat. 27.83		1.444513
To distance 28.31		1.451917

EXAMPLE 4.

Sta.	Bearing.	Dist.	N.	S.	E.	W.
1	S. 10° E.	92.20		90.80	16.01	
2	S. 15 W.	120.50		116.39		31.19
3	S. 18½ W.	205.00		194.40		65.05
4	S. 71½ E.	68.00		21.58	64.49	
5						
				423.17	80.50	96.24
						80.50
						15.74

Then,

As diff. of latitude 423.17	Ar. Co.	7.373485
Is to departure 15.74		1.197005
So is radius		10.000000
To tang. bearing 2° 8′		8.570490

And,

As cosine bearing 2° 8′	Ar. Co.	0.000301
Is to radius		10.000000
So is diff. lat. 423.17		2.626515
To distance 423.46		2.626816

PROBLEM II.

EXAMPLE 2.

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.
1	S. 40½ E.	N. 85½ E.	31.80	2.49		31.70	
2	N. 54 E.	North.		(2.08)			
3	N. 29¼ E.	N. 24¾ W.	2.21	2.01			.93
4	N. 28¾ E.	N. 25¼ W.	35.35	31.98			15.08
5	N. 57 W.	S. 69 W.			(7.49)		(19.51)
6	S. 47 W.	S. 7 E.	31.30		31.07	3.82	
				38.56	38.56	35.52	35.52

As sine changed bearing 69° - - - - Ar. Co. 0.029848
Is to radius - - - - - - - - - - - - - - 10.000000
So is departure 19.51 - - - - - - - - - - - - 1.290257
To distance 5th side 20.90 - - - - - - - - - 1.320105

And, As radius - - - - - - - - - - - - Ar. Co. 0.000000
Is to cotang. bearing 69° - - - - - - - - - 9.584177
So is departure 19.51 - - - - - - - - - - - - 1.290257
To diff. latitude 7.49 S. - - - - - - - - - - 0.874434

PROBLEM III.

EXAMPLE 2.

Sta.	Bearing.	Changed Bearing.	Dist.	N.	S.	E.	W.
1	S. 40½ E.	N. 85½ E.	31.80	2.49		31.70	
2	N. 54 E.	North.		(2.09)			
3	N. 29¼ E.	N. 24¾ W.	2.21	2.01			.93
4	N. E.		35.35	(31.97)			(15.08)
5	N. 57 W.	S. 69 W.	20.90		7.49		19.51
6	S. 47 W.	S. 7 E.	31.30		31.07	3.82	
				38.56	38.56	35.52	35.52

Then,

As distance 4th side 35.35 - - - -	Ar. Co.	8.451611
Is to departure 15.08 - - - - - - - - - -		1.178401
So is radius - - - - - - - - - - - -		10.000000
To sine chang. bearing N. 25° 15′ W. - -		9.630012
54		
Bearing 4th side N. 28° 45′ E.		

And,

As radius - - - - - - - - - - - -	Ar. Co.	0.000000
Is to cos. chang. bearing 25° 15′ - - - - -		9.956387
So is distance - - - - - - - - - - - -		1.548389
To diff. latitude 31.97 - - - - - - - - -		1.504776

PROBLEM IV.

EXAMPLE 2. (Pl. 1, fig. 13.)

	Bearing.	Dist.	N.	S.	E.	W.
FA	S. E.	31.80				
AB	N. 54 E.	2.08	1.23		1.68	
BC	N. 29¼ E.	2.21	1.92		1.08	
CD	N. 28¾ E.	35.35	31.00		17.00	
DE	N. 57 W.	20.90	11.38			17.52
EF	S. W.	31.30				
Diff. latitude of EA			45.53		19.76	17.52
					17.52	
				Departure of EA	2.24	

Then, As diff. lat. EA 45.53 - - - - - Ar. Co. 8.341702
Is to departure 2.24 - - - - - - - - - 0.350248
So is radius - - - - - - - - - - - - 10.000000

To tang. bearing EA 2° 49′ - - - - - 8.691950

And,

As cosine bearing 2° 49′	Ar. Co.	0.000525
Is to radius		10.000000
So is diff. lat.		1.658298
To distance EA 45.59		1.658823

To find AEF:

AF	31.80		
AE	45.59	Ar. Co.	8.341177
EF	31.30	Ar. Co.	8.504456
	2)108.69		
Half sum	54.34		1.735120
Difference	22.54		1.352954
			2)19.933707
Cos. ½ AEF	22° 6′		9.966853
AEF	44° 12′		
Bearing of EA	2° 49′		
" EF	S. 47° 1′ W.		

To find EAF and bearing of FA:

As AF 31.80	Ar. Co.	8.497573
Is to EF 31.30		1.495544
So is sin. AEF 44° 12′		9.843336
To sin. EAF 43° 20′		9.836453
Bearing of EA 2° 49′		
" AF 40° 31′		

CHAPTER III.

CONTENT OF LAND.

PROBLEM I.

EXAMPLE 4.

Here, Area = 176.4 × 176.4 = 31116.96 Sq. Perches,
= 194 A. 1 R. 36.96 P.

EXAMPLE 5.

Here, Area = $52.25 \times 38.24 = 1998.04$ Sq. Ch.
= 199 A. 3 R. 8.64 P.

EXAMPLE 6.

Here, Area = $16.54 \times 12.37 = 204.5998$ Sq. Ch.
= 20 A. 1 R. 33.5968 P.

EXAMPLE 7.

Here, Area = $21.16 \times 11.32 = 239.5312$ Sq. Ch.
= 23 A. 3 R. 32.4992 P.

PROBLEM 2.

EXAMPLE 2

Here, Area = $\frac{18.37 \times 13.44}{2} = \frac{246.8928}{2} = 123.4464$ Sq. Ch.
= 12 A. 1 R. 15.1424 P.

EXAMPLE 3.

Here, Area = $\frac{49 \times 34}{2} = \frac{1666}{2} = 833$ Sq. Pe.
= 5 A. 0 R. 33 Pe.

PROBLEM 3.

EXAMPLE 2. (Pl. 1, fig. 1.)

As radius - - - - - - - - - -		Ar. Co.	0.000000
Is to sin. A 47° 30' - - - - - - - - - -			9.867631
So is AB × AC	AB 15.36 - - - - - - -		1.186391
	AC 11.46 - - - - - - -		1.059185
To double area 129.78 - - - - - - - - -			2.113207

ABC - 64.89 Ch. = 6 A. 1 R. 38. 24 P

Example 3. (Pl. 1, fig. 14.)

Here, As radius		Ar. Co.	0.000000
Is to sin. A 66° 30′			9.962398
So is AB × AC	AB 13.84		1.141136
	AC 18.23		1.260787
To 2 ABC	231.38		2.364321

ABC - 115.69 Ch. = 11 A. 2 R. 11.04 P.

Example 4. (Pl. 1, fig. 15.)

Here, As radius		Ar. Co.	0.000000
Is to sin. A 121° 45′			9.929599
So is AB, AC	AB 19.74		1.295347
	AC 17.34		1.239049
To 2 ABC	291.07		2.463995

ABC - 145.535 Ch. = 14 A. 2 R. 8.56 P.

PROBLEM 4.

Example 2. (Pl. 1, fig. 1.)

Here, Angle C = 180—(A+B) = 43°. Hence,

As rad., sin. C	radius	Ar. Co.	0.000000
	sin. C 43°	Ar. Co.	0.166217
Is to sin. A, sin. B	sin. A 63°		9.949881
	sin. B 74°		9.982842
So is AB^2	AB 24.32		1.385964
	AB		1.385964
To 2 ABC	742.8		2.870868

ABC - 371.4 Ch. = 37 A. 0 R. 22.4 P.

Example 3.

Here, the angle C = 94° 15′. Hence

As rad., sin. C	rad.	Ar. Co.	0.000000
	sin. C 94° 15′	Ar. Co.	0.001196
Is to sin. A, sin. B	sin. A 37° 30′		9.784447
	sin. B 48° 15′		9.872772
So is AB^2	AB 17.36		1.239550
	AB		1.239550
To 2 ABC	137.25		2.137515

ABC - 68.625 Ch. = 6 A. 3 R. 18 P.

PROBLEM 5.

EXAMPLE 2.

Here, $10.64+12.28+9.00 = 31.92$ = sum of sides.

Half sum	15.96	log.	1.203033
Remainders	5.32		0.725912
	3.68		0.565848
	6.96		0.842609
			2)3.337402
Area	10)46.63 Ch.		1.668701

4.663 = 4 A. 2 R. 26.08 P.

EXAMPLE 3.

Here, $20+30+40 = 90$ = sum of sides.

Half sum	45	1.653213
Remainders	25	1.397940
	15	1.176091
	5	0.698970
		2)4.926214
	10)290.47	2.463107

29.047 A. = 29 A. 0 R. 7.52 P.

PROBLEM 6.

EXAMPLE 2.

Here, $16.10 \times \frac{6.80+3.40}{2} = 16.1 \times 5.1 = 82.11$ Ch.

= 8 A. 0 R. 33.76 P.

EXAMPLE 3.

Here, $24 \times \frac{8.27+12.43}{2} = 24 \times 10.35 = 248.4$ Ch.

= 24 A. 3 R. 14.4 P.

PROBLEM 7.

EXAMPLE 2.

$$\text{Here, Area} = \frac{12.41+8.22}{2} \times 5.15 = 53.12225 \text{ Ch.}$$
$$= 5 \text{ A. } 1 \text{ R. } 9.956 \text{ P.}$$

EXAMPLE 3.

$$\text{Here, Area} = \frac{11.34+18.46}{2} \times 13.25 = 197.425 \text{ Ch.}$$
$$= 19 \text{ A. } 2 \text{ R. } 38.8 \text{ P}$$

PROBLEM 9.

EXAMPLE 2.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	N. 23° 45′ W.	6.46	5.91			2.60	0	0	5.91			2.60	2.60	15.3660	
2	N. 65° 16′ E.	4.40	1.84		4.00		0	0	1.84		4.00		4.00	7.3600	
3	S. 41° 49′ E.	9.68		7.21	6.45		1	1		7.22	6.44		14.44		104.2568
4	S. 66° 22′ W.	4.895		1.96		4.48	0	0		1.96		4.48	16.40		32.1440
5	N. 24° 2′ W.	2.625	2.40			1.07	0	0	2.40			1.07	10.85	26.0400	
6	S. 66° 59′ W.	2.49		.97		2.29	0	0		.97		2.29	7.49		7.2653
		30.550	10.15 10.14	10.14	10.45 10.44	10.44	1	1	10.15	10.15	10.44	10.44		48.7660	143.6661 48.7660

Error S. .01 .01 Error W.

2)94.9001

47.4500

4.745
4

2.980
40

39.200

Area, 4 A. 2 R. 39.2 P.

EXAMPLE 3.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. N.	Cor. W.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	S. 62° W.	7.57		3.55		6.68	1	1		3.54		6.69	24.61		87.1194
2	N. 43½ W.	5.89	4.28			4.05	1	1	4.29			4.06	13.86	59.4594	
3	North.	5.82	5.82				1	1	5.83			.01	9.79	57.0757	
4	N. 33½ W.	8.83	7.36			4.88	1	1	7.37			4.89	4.89	36.0393	
5	N. 48 E.	4.81	3.22		3.57		0	1	3.22		3.56		3.56	11.4632	
6	N. 12 E.	4.66	4.56		.97		0	0	4.56		.97		8.09	36.8904	
7	N. 62½ E.	5.27	2.43		4.68		0	1	2.43		4.67		13.73	33.3639	
8	S. 6½ E.	5.60		5.57	.64		1	1		5.56	.63		19.03		105.8068
9	S. 40½ E.	5.87		4.46	3.82		1	1		4.45	3.81		23.47		104.4415
10	East.	6.54			6.54		1	1	.01		6.53		33.81	.3381	
11	North.	5.52	5.52				1	1	5.53			.01	40.33	223.0249	
12	N. 68¼ E.	3.10	1.15		2.88		0	0	1.15		2.88		43.20	49.6800	
13	S. 30 E.	7.90		6.84	3.95		1	1		6.83	3.94		50.02		341.6366
14	S. 23 W.	8.80		8.10		3.44	1	1		8.09		3.45	50.51		408.6259
15	S. 31½ E.	6.42		5.48	3.35		1	1		5.47	3.34		50.40		275.6880
16	S. 50 W.	8.40		5.40		6.44	1	1		5.39		6.45	47.29		254.8931
17	N. 44 W.	6.85	4.93			4.76	1	1	4.94			4.77	36.07	178.1858	
			39.27	39.41	30.40	30.25								685.5207	1578.2113
				39.27	30.25										685.5207
		Error S.		13	15	Error E.									2)892.6906
															446.3453
															44.63453
															4
															2.53812
															40
															21.52480

Area, 44 A. 2 R. 21.525 P.

EXAMPLE 4.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	S. 85¼ E.	23.30		1.92	23.22		73.36		140.8512
2	S. 19 E.	31.12		29.42	10.13		106.71		3139.4082
3				(11.24)		(25.71)	91.13		1024.3012
4	N. 64 W.	29.72	13.03			26.71	38.71	504.3913	
5	N. 15½ W.	22.46	21.64			6.00	6.00	129.8400	
6	N. 58 E.	25.94	13.75		22.00		22.00	302.5000	
7	S. 27¾ E.	6.60		5.84	3.07		47.07		274.8888
			48.42	48.42	58.42	58.42		936.7313	4579.4494
									936.7313
									2)3642.7181
									1821.35905
									182.135905
									4
									.543620
									40
									21.744800

As diff. lat. 11.24 - - - - - - - - - Ar. Co. 8.949234
Is to departure 25.71 - - - - - - - - - - - 1.410102
So is radius - - - - - - - - - - - - - - 10.000000
To tang. bearing 66° 23′ - - - - - - - - - 10.359336

And,

As cosine bearing 66° 23′ - - - - - - Ar. Co. 0.397272
Is to radius - - - - - - - - - - - - - - 10.000000
So is diff. lat. 11.24 - - - - - - - - - - 1.050766
To distance 28.06 - - - - - - - - - - - - 1.448038

EXAMPLE 5. (Pl. 2, fig. 4.)

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor.	Cor.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	S. 60¾ W.	10.34		5.06		9.02				5.06		9.02	25.38		128.4228
2	N. 27¼ W.	17.88	15.89			8.18			15.89			8.18	8.18	129.9802	
3	N. 51 E.	15.85	9.97		12.32				9.97		12.32		12.32	122.8304	
4	N. (8¼) E.	9.61	(9.51)		(1.38)				9.51		1.38		26.02	247.4502	
5	S. (73°) E.	19.18		(5.61)	(18.34)					5.61	18.34		45.74		256.6014
6	S. 16¾ E.	22.21		21.27	6.40		1	1		21.26	6.41		70.49		1498.6174
7	S. 71½ W.	16.66		5.29		15.80				5.29		15.80	61.10		323.2198
8	N. 71¼ W.	5.76	1.85			5.45			1.85			5.45	39.85	73.7225	
			27.22	37.23	38.44	38.45								573.9833	2206.8614
															573.9833
															2)1632.8781
															816.43905
															81.643905
															4
															2.575620
															40
															23.02480

Area 81 A. 2 R. 23.0248 P.

As diff. lat. DF 3.91 - - - - - - Ar. Co. 9.407823
Is to depart. 19.73 - - - - - - - - - - - 1.295127
So is radius - - - - - - - - - - - - - - 0.000000

To tang. bearing N. 78° 47′ E. - - - - - - 10.702950

As cosine bearing 78° 47′ - - - Ar. Co. 0.711036
Is to radius - - - - - - - - - - - - - - 10.000000
So is diff. latitude 3.91 - - - - - - - - - 0.592177

To distance DF 20.10 - - - - - - - - - 1.303213

FE 19.18
ED 9.61 - - - - - - - Ar. Co. 9.017277
DF 20.10 - - - - - - - - " " 8.696804

2)48.89

Half sum 24.445 - - - - - - - - - - - - 1.388190

Diff. 5.265 - - - - - - - - - - - 0.721398

2)19.823669

Cos. ½ FDE 35° 17′ - - - - - - - 9.911834

FDE 70° 34′
Bearing DF N. 78 47 E.

Bearing DE N. 8° 13′ E.

As FE 19.18 - - - - - - - - - - - - - - 8.717151
Is to DE 9.61 - - - - - - - - - - - - - - 0.982723
So is sin. FDE 70° 34′ - - - - - - - - - 9.974525

To sin. DFE - - 28° 12′ - - - - - - - 9.674399

Bearing FD S. 78 47 W.

106 59
180

Bearing EF S. 73 1 E.

EXAMPLE 6. (Fig. 81, *Surveying.*)

Sta.	Bearing.	Changed bearing.	Dist.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
AB	N. $51\frac{1}{4}$ W.	North.		(26.47)				30.12	797.2764	
BC	S. $45\frac{1}{2}$ W.	N. $83\frac{1}{4}$ W.	15.16	1.78			15.06	15.06	26.8068	
CD	N. 50 W.	N. $1\frac{1}{4}$ E.	22.10	22.09		.48		.48	10.6032	
DE	North.	N. $51\frac{1}{4}$ E.	18.83	11.79		14.69		15.65	184.5135	
EF	N. 48 E.	S. $80\frac{3}{4}$ E.	22.60		3.64	22.30		52.64		191.6096
FG	N. $25\frac{1}{2}$ W.	N. $25\frac{3}{4}$ E.	20.17	18.16		8.76		83.70	1519.9920	
GH	East.	S. $38\frac{3}{4}$ E.	26.57		20.72	16.63		109.09		2260.3448
HI	S. $30\frac{1}{2}$ E.	S. $20\frac{3}{4}$ W.	22.86		21.37		8.09	117.63		2513.7531
IK	S. 44 W.	N. $84\frac{3}{4}$ W.	15.04	1.37			14.98	94.56	129.5472	
KL	S. 47 E.	S. $4\frac{1}{4}$ W.	28.55		28.47		2.12	77.46		2205.2862
LA	S. $20\frac{1}{2}$ W.	S. $71\frac{3}{4}$ W.			(7.46)		(22.61)	52.73		393.3658
				81.66	81.66	62.86	62.86		2668.7391	7564.3595
										2668.7391
										2)4895.6204
										2447.8102
										244.78102
										4
										3.12408
										40
										4.96320

Area 244 A. 3 R. 4.9632 P.

As sine changed bearing LA 71° 45′ - - Ar. Co. 0.022414
Is to radius - - - - - - - - - - - - - - - - 10.000000
So is departure 22.61 - - - - - - - - - - - - 1.354301

To distance LA 23.81 - - - - - - - - - - - 1.376715

And,

As radius - - - - - - - - - - - - - Ar. Co. 0.000000
Is to cotang. bearing - - - - - - - - - - - - 9.518184
So is departure - - - - - - - - - - - - - - 1.354301

To diff. latitude 7.46 - - - - - - - - - - - 0.872485

EXAMPLE 7. (Fig. 80, *Surveying.*)

To find the third side.

Sta.	Bearing.	Dist.	N.	S.	E.	W.
EA	S. 52 W.	10.70		6.59		8.43
AB	S. 7½ W.	13.92		13.80		1.82
BC	S. 33¼ E.	9.00		7.53	4.93	
				27.92		10.25
						4.93
						5.32

As diff. lat. EC 27.92 - - - - - - - - Ar. Co. 8.554085
Is to depart. 5.32 - - - - - - - - - - - - - 0.725912
So is radius - - - - - - - - - - - - - - - - 10.000000

To tang. bearing S. 10° 47′ W. - - - - - - - 9.279997

As cosine bearing 10° 47′ - - - - - - Ar. Co. 0.007737
Is to radius - - - - - - - - - - - - - - - - 10.000000
So is diff. lat. - - - - - - - - - - - - - - 1.445915

To distance 28.42 - - - - - - - - - - - - - 1.453652

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	North.	7.81	7.81				2	1	7.79			.01	33.09	257.7711	
2	S. 76¼ E.	18.15		4.32	17.63		3	3		4.35	17.60		50.68		220.4580
3	S. 10¾ W.	28.42		27.92		5.30	5	5		27.97		5.35	62.93		1760.1521
4	N. 84¼ W.	27.12	2.72			26.98	5	4	2.67			27.02	30.56	81.5952	
5	N. 4½ W.	22.00	21.93			1.73	4	4	21.89			1.77	1.77	38.7453	
6	East.	16.58			16.58		3	3		.03	16.55		16.55		.4965
		120.08	32.46	32.24	34.21	34.01								378.1116	1981.1066 378.1116
															2)1602.9950
															801.4975 4
															.59900 40
															23.96

Area, 80 A. 0 R. 23.96 P.

PROBLEM 10.

EXAMPLE 2.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
1	N. 36¾ W.	30.00	24.04			17.95	1	1	24.03			17.96	17.96	431.5788	
2	N. 56¼ E.	21.60	12.00		17.96		1	0	11.99		17.96		17.96	215.3404	
3	N. 26½ E.	13.44	12.03		6.00		0	0	12.03		6.00		41.92	504.2976	
4	S. 71½ E.	18.96		6.02	17.98		0	0		6.02	17.98		65.90		396.7180
5	S. 26½ E.	13.46		12.04	6.01		0	0		12.04	6.01		89.89		1082.2756
6	S. 45 W.	42.41		29.98		29.98	1	1		29.99		29.99	65.91		1976.6409
		139.87	48.07	48.04	47.95	47.93	3	2						1151.2168	3455.6345
			48.04		47.93										1151.2168
			Error S. .03		.02 Error W.										2)2304.4177
															1152.2088
															134.4907
															1286.6995
															128.66995
															4
															2.67980
															40
															27.19200

128 A. 2 R. 27.19 P.

1st Stat. Line.

No.	Dist.	Off-sets.	Int. Dist.	Sums.	Areas.
1	0.00	0.50			
2	6.10	3.40	6.10	3.90	23.7900
3	10.15	3.10	4.05	6.50	26.3250
4	14.08	3.96	3.93	7.06	27.7458
5	19.20	2.70	5.12	6.66	34.0992
6	21.60	0.55	2.40	3.25	7.8000
					2)119.7600
					59.8800

2d Stat. Line.

No.	Dist.	Off-sets.	Areas.
1	0·00	0.55	
2	13.44	0.55	7.3920
			7.3920

3d Stat. Line.

No.	Dist.	Off-sets.	Int. Dist.	Sums.	Areas.
1	0.00	0.55			
2	4.20	2.50	4.20	3.05	12.8100
3	8.05	3.20	3.85	5.70	21.9450
4	15.15	2.45	7.10	5.65	40.1150
5	18.96	0.50	3.81	2.95	11.2395
					2)86.1095
					43.0547

4th Stat. Line.

No.	Dist.	Off-sets.	Int. Dist.	Sums.	Areas.
1	0.00	0.50			
2	5.12	2.75	5.12	3.25	16.6400
3	10.00	1.90	4.88	4.65	22.6920
4	13.46	0.70	3.46	2.60	8.9960
					2)48.3280
					24.1640

1st Stat. Line,	59.8800
2d " "	7.3920
3d " "	43.0547
4th " "	24.1640
Area off-sets,	134.4907

Problem 10. (Example 3.)

Adding 1′ to each of the angles, we find the bearings as follows:

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
D	N.	8.35	8.35				2	8.33						
C	N. 67° 34′ E.	2.41	.92		2.23		0	.92		2.23		2.23	2.0516	
B	S. 21° 8′ E.	9.42		8.79	3.40		2		8.81	3.40		7.86		69.2466
A	S. 85° 39′ W.	5.65		.43		5.63	1		.44		5.63	5.63		2.4772
			9.27	9.22	5.63	5.63	5	9.25	9.25	5.63	5.63		2.0516	71.7238
			.22											2.0516
			.05 Error S.											2)69.6722
														34.8361
														4.4667
														39.3028
														3.93028
														4
														3.72112
														40
														28.8448

Area, 3 A. 3 R. 28.8448 P.

On AB.

Dist.	Off-sets.	Int. Dist.	Sums.	Areas.
0.00	0.00			
1.00	.35	1.00	.35	.3500
1.59	.26	.59	.61	.3599
2.25	.50	.66	.76	.5016
4.17	.46	1.92	.96	1.8432
4.58	.54	.41	1.00	.4100
7.32	.16	2.74	.70	1.9180
7.66	.06	.34	.22	.0748
8.25	.11	.59	.17	.1003
8.75	.00	.50	.11	.0550
9.00	.21	.25	.21	.0525
9.42	.00	.42	.21	.0882
				2)5.7535
				2.8767

On CD.

Dist.	Off-sets.	Int. Dist.	Sums.	Areas.
0.00	0.00			
.34	.41	.34	.41	.1394
1.00	.00	.66	.41	.2706
2.15	.68	1.15	.68	.7820
3.75	.00	1.60	.68	1.0880
4.40	.00	.65	.00	.0000
6.50	.25	2.10	.25	.5250
8.00	.00	1.50	.25	.3750
				2)3.1800
				1.5900
				2.8767
				4.4667

PROBLEM 13.

EXAMPLE 2. (Pl. 2, fig. 5.)

Here the various angles will be found to be as in the following proportions. Then,

To find log. of GA:

As sin. FAG 88° 30′	Ar. Co.	0.000149
Is to sin. GFA 68° 30′		9.968678
So is FG 20 ch.		1.301030
To GA		1.269857

To find log. GB:

As sin. FBG 42°	Ar. Co.	0.174489
Is to sin. GFB 24°		9.609313
So is FG		1.301030
To GB		1.084832

To find log. GC:

As sin. GCF 43° 15′	Ar. Co.	0.164193
Is to sin. GFC 38°		9.789342
So is FG		1.301030
To GC		1.254565

To find log. GD:

As sin. GDF 44° 30′	Ar. Co.	0.154338
Is to sin. GFD 59°		9.933066
So is GF		1.301030
To GD		1.388434

To find log. GE:

As sin. GEF 35° 30′	Ar. Co.	0.236046
Is to sin. GFE 103° 30′		9.987832
So is GF		1.301030
To GE		1.524908

To find 2 ABG:

As radius	Ar. Co.	0.000000
Is to sin. AGB 91°		9.999934
So is BG		1.084832
AG		1.269857
To 2 ABG 226.268		2.354623

To find 2 BGC:

As radius	Ar Co.	0.000000
Is to sin. BGC 15° 15′		9.420007
So is GB		1.084832
GC		1.254565
To 2 BGC 57.465		1.759404

To find 2 CGD:

As radius	Ar. Co.	0.000000
Is to sin. CGD 22° 15′		9.578236
So is GC		1.254565
GD		1.388434
To 2 CGD 166.431		2.221235

To find 2 DGE:

As radius	Ar. Co.	0.000000
Is to sin. DGE 35° 30′		9.763954
So is GD		1.388434
GE		1.524908
To 2 DGE 475.657		2.677296

To find 2 EGA:

As radius	Ar. Co.	0.000000
Is to sin. EGA 18°		9.489982
So is EG		1.524908
GA		1.269857
To 2 EGA	192.640	2.284747

2 DGE	475.657
2 CGD	166.431
2 BGC	57.465
	892.193
2 AGB	226.268
	2)665.925
ABCDE	332.9625 Ch. = 33 A. 1 R. 7.4 P.

CHAPTER IV.

LAYING OUT AND DIVIDING LAND.

PROBLEM 1.

EXAMPLE 2.

Here, 325 Acres = 3250 chains.
And side = $\sqrt{3250}$ = 57 chains.

PROBLEM 2.

EXAMPLE 2.

Here breadth = $\frac{5 \text{ Acres}}{8 \text{ chains}} = \frac{50}{8} = 6.25$ chains.

PROBLEM 3.

EXAMPLE 2.

Here, 27 A. 3 R. 20 P. = 4460 P.
And, As 7 : 9 : : 4460 : 5734.2857.
$\sqrt{5734.2857}$ = 75.725 = length.
Also, As 9 : 7 : : 75.725 : 58.897 = breadth.

PROBLEM 4.

EXAMPLE 2. (Pl. 2, fig. 6.)

Here, 114 A. 2 R. 33.4 P. = 1147.0875 chains.
Also, $\sqrt{1147.0875+7.55^2} = \sqrt{1204.09} = 34.7$.
And, 34.7+7.55 = 42.25 length.
34.7—7.55 = 27.15 breadth.

PROBLEM 5.

EXAMPLE 3. (Pl. 2, fig. 7.)

Here, 2 Acres = 320 Perches.

And,

As AB, sin. A	AB 30 P.	Ar. Co.	8.522879
	sin. A 71° 15′	Ar. Co.	0.023682
Is to 2 ABC 640			2.806180
So is radius			10.000000
To AC 22.53			1.352741

EXAMPLE 4. (Pl. 2, fig. 8.)

As AB, sin. A	AB 32.26	Ar. Co.	8.491336
	sin. A 83° 30′	Ar. Co.	0.002801
Is to ABCD 740			2.869232
So is radius			10.000000
To AD 23.09			1.363369

PROBLEM 6.

EXAMPLE 2. (Pl. 2, fig. 9.)

Here, 27 A. 1R. 16 P. = 273.5 Ch.

And,

As ABC 273.5	Ar. Co.	7.563043
Is to BDC 100		2.000000
So is AB 35.20		1.546543
To BD 12.87		1.109586

PROBLEM 7.

EXAMPLE 2. (Pl. 2, fig. 10.)

Construction.

Make AB, equal to the greater of the given sides (20). Draw BD perpendicular to AB, equal to twice the given area, divided by AB (12.39). Through D draw DC parallel to AB. Then if AC be made equal to the other given side (16.25), and BC be joined; ABC will be the triangle.

For the Division Line. Make AP = 8.50 the given distance. Take AF to AC in the ratio of the part to be cut off to the whole area. Join PF, draw BG parallel to it; then PG will be the division line.

Demonstration.

AB : AP : : AG : AF, Therefore, AB. AC : AP. AG :: AC . AG : AG . AF : : AC : AF, or AB . AC . sin. A : AP . AG . sin. A : : AC : AF : : m : n (m being the whole area, and n the part to be cut off.) Hence, since AC . AB sin. A = m, AP . AG sin. A = n, and PG is the division line.

Calculation.

As ABC 123.9375 - - - - - - -	Ar. Co.	7.906798
Is to APG 30 - - - - - - - - - - - -		1.477121
So is AB . AC { AB 20 - - - - - - -		1.301030
{ AC 16.25 - - - - -		1.210853
To AP . AG - - - - - - - - - - - -		1.895802
AP = 8.50 - - - - - - - - - -		0.929419
AG = 9.255 - - - - - - - - - -		0.966383

PROBLEM 8.

EXAMPLE 2. (Pl. 2, fig. 11.)

Here, As BAC 100 ch. - - - - - - -	Ar. Co.	8.000000
Is to BDG 45 - - - - - - - - - - - -		1.653213
So is BA^2 { BA 25 - - - - - - -		1.397940
{ BA - - - - - - - -		1.397940
To BD^2 - - - - - - - - - - - -		2)2.449093
BD = 16.77 - - - - - - - - - - -		1.224546

PROBLEM 9.

EXAMPLE 2. (Pl. 2, fig. 12.)

Here the angles are, A = 71° 45′, B = 49° 15′, and C = 59°. Hence,

As sin. A . sin. B { sin. A 71° 45′	Ar. Co.	0.022414
{ sin. B 49° 15′	" "	0.120580
Is to rad. . sin. C { radius - - - - - - -		10.000000
{ sin. C 59° - - - - -		9.933066
So is 2 ABC 80 ch. - - - - - - - - -		1.903090
To AB^2 - - - - - - - - - - - -		2)1.979150
AB = 9.763 - - - - - - - - - - -		.989575

PROBLEM 10.

EXAMPLE 2. (Pl. 2, fig. 13.)

Here the angles A = 99° 30′, B = 122°, and P = 41° 30′.

And, As sin. A. sin. B	sin. A 99° 30′	Ar. Co.	0.005997
	sin. B 122°	" "	0.071580
Is to rad. . sin. P	radius		10.000000
	sin. P 41° 30′		9.821265
So is 2 ABCD	50 ch.		1.698970
To fourth term	39.61		1.597812

AB^2 - - 36

CD^2 - $\sqrt{75.61} = 8.695$.

Also, As sin. P	41° 30′	Ar. Co.	0.178735
Is to sin. B	122°		9.928420
So is DC—AB	2.695		0.430559
To AD	3.449		0.537714

EXAMPLE 3. (Pl. 3, fig. 1.)

Here the angles are, A = 90°, B = 73° 30′, and P = 16° 30′.

Also, As sin. A. sin. B	sin. A 90°	Ar. Co.	0.000000
	sin. B 73° 30′	" "	0.018263
Is to rad. . sin. P	radius		10.000000
	sin. P 16° 30′		9.453342
So is 2 ABCD	160 ch.		2.204120
To fourth term	47.39		1.675725

AB^2 - 182.25

CD = $\sqrt{134.86} = 11.61$.

And, As sin. P	16° 30′	Ar. Co.	0.546658
Is to sin. B	73° 30′		9.981737
So is AB—CD	1.89		0.276462
To AD	6.38		0.804857

PROBLEM 12.

EXAMPLE 2. (Pl. 3, fig. 6.)

Here, As $2 : 1 :: BC^2(100) : EF^2 = 50$,

$EF = \sqrt{50} = 7.07$.

And, As BC (10) : EF (7.07) :: AB (15) : AF = 10.605.

PROBLEM 13.

EXAMPLE 2. (Pl. 3, fig. 7.)

Here the angles are, A = 36° 30′, B = 100° 30′, C = 43°, E = 74° 30′, and F = 69°.

As sin. E . sin. F	sin. E 74° 30	Ar. Co.	0.016089
	sin. F 69 - -	" "	0.029848
Is to sin. C . sin. B	sin. C 43° - - - - -		9.833783
	sin. B 100° 30′ - - -		9.992666
So is BC^2	BC 18.66 - - - - -		1.270912
	BC - - - - - - - -		1.270912
To fourth term	259.54 - - - - - - - -		2.414210

4

9)1038.16

$EF = \sqrt{115.35} = 10.74$.

As sin. A 36° 30′ - - - - - -	Ar. Co.	0.225612
Is to sin. E 74° 30′ - - - - - - - - -		9.983911
So is EF 10.74 - - - - - - - - - - -		1.031004
To AF 17.40 - - - - - - - - - - -		1.240527

PROBLEM 14.

EXAMPLE 2. (Pl. 3, fig. 8.)

Here, $EF = \sqrt{\left(\frac{AB^2 + CD^2}{2}\right)} = \sqrt{5796.18} = 76.13$.

And, DC—AB (29.4) : FE—AB (16.13) :: AD (30) : AF = 16.46

PROBLEM 16.

Example 1. (Pl. 3, fig. 12.)

The area of this tract may be found to be 858.552 square chains. (The latitudes and departures are mostly given in the subsequent operations.)

To find area ABCDE, and the latitude and departure of EA.

	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
AB		9.15		6.46	40.86		373.8690
BC	17.21			17.20	17.20	296.0120	
CD	10.41		2.89		2.89	30.0849	
DE		3.61	15.60		21.38		77.1818
EA		(14.86)	(5.17)		42.15		626.3490
	27.62	27.62	23.66	23.66		326.0969	1077.3998 326.0969
							751.3029
						2 ABCDEI	858.552
							2)107.2491
						AEI	53.6245

	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
AE	14.86			5.17	5.17	76.8262	
EF	.93		21.49		21.49	19.9857	
FA		(15.79)		(16.32)	26.66		420.9614
						96.8119	96.8119
							2)324.1495
						AEF	162.07475

As AEF 162.07475 - - - - - Ar. Co. 7.790285
Is to AEI 53.6245 - - - - - - - - - - 1.729363
So is lat. EF .93 - - - - - - - - - —1.968483

To lat. EI .31 - - - - - - - - - - —1.488131

As AEF - - - - - - - - - - Ar. Co. 7.790285
Is to AEI - - - - - - - - - - - - - 1.729363
So is depart. EF 21.49 - - - - - - - - 1.332236

To depart. EI 7.11 - - - - - - - - - - 0.851884

	N.	S.	E.	W.
AE	14.86			5.17
EI	.31		7.11	
IA		(15.17)		(1.94)

As diff. lat. 15.17 - - - - - - Ar. Co. 8.819014
Is to depart. 1.94 - - - - - - - - - - 0.287802
So is radius - - - - - - - - - - - - 10.000000

To tang. bearing AI 7° 17′ - - - - - - - 9.106816

As cos. bearing - - - - - - - Ar. Co. 0.003518
Is to radius - - - - - - - - - - - - 10.000000
So is diff. lat. 15.17 - - - - - - - - - 1.180986

To dist. AI 15.29 - - - - - - - - - - 1.184504

CHAPTER V.

MISCELLANEOUS QUESTIONS.

QUESTION 1.

Here $\frac{1}{2}$ Acre = 2420 square yards;

And radius = $\sqrt{\left(\frac{2420}{3.1416}\right)} = \sqrt{770.3081} = 27.75$.

QUESTION 2.

Construction.

Make AB (Pl. 4, fig. 1) = 40 = one of the given sides, and at A draw AL perpendicular to AB and = $\frac{320}{40} = 8$; through L draw GH parallel to AB, and with the centre A and distance = 20 = the other given side, describe an arc, cutting GH in D and C; join AC, BC, AD, and BD: then will ABC and ABD answer the conditions of the question.

Calculation.

AE = AF = $\sqrt{AC^2 - CE^2} = \sqrt{400 - 64} = 18.3303$; and BE = AB—AE = 21.6697; therefore, BC = $\sqrt{BE^2 + EC^2} = \sqrt{533.57589809}$ = 23.099. Also, BF = AB + AF = 58.3303, and BD = $\sqrt{BF^2 + FD^2}$ = $\sqrt{3466.42389809}$ = 58.876.

Another Solution.

Find AE = AF as before. Then, from Geometry, $BC^2 = AB^2 + AC^2 - 2\ AB.AE = 2000 - 1466.424 = 533.576$, and BC = 23.099. Also, $BD^2 = AB^2 + AD^2 + 2\ AB.AF = 2000 + 1466.424 = 3466.424$, and BF = 58.876.

QUESTION 3.

Here it is evident the number of acres will be inversely as the number of square yards in a Perch:

Therefore, $6^2 : 5.5^2 :: 110$ A. : 92 A. 1 R. $28\frac{8}{9}$ P. Cheshire.

And $7^2 : 5.5^2 :: 110$ A. : 67 A. 3 R. $25\frac{15}{49}$ P. Irish.

QUESTION 4.

Here $\frac{28}{12} = \frac{7}{3}$ = twice the thickness of the wall, also 840 links = 554.4 feet = the longer diameter within the walls; 612 links = 403.92 feet = the shorter; $554.4 + \frac{7}{3} = \frac{1670.2}{3}$ = longer diameter outside, and $403.92 + \frac{7}{3} = \frac{1218.76}{3}$ = shorter. By Prob. 10, Chap. III. the area within the walls = $554.4 \times 403.92 \times .7854 = 223933.248 \times .7854 =$ 175877.1729792 ft. = 4 A. 0 R. 6 P. The area to the outside = $\frac{1670.2}{3} \times \frac{1218.76}{3} \times .7854 = \frac{2035572.952}{9} \times .7854 = \frac{1598738.9965008}{9}$ = 177637.6662778 feet. Therefore 177637.666 — 175877.173 = 1760.493 = area the wall stands upon.

QUESTION 5.

Here the area of an ellipse whose diameters are 3 and 2 is 4.7124. Then, since similar figures are as the squares of their like dimensions, we have, As 4.7124 : 160 :: 9 : 305.5768 = square of the longer diameter; consequently $\sqrt{305.5768} = 17.481$ = longer diameter; and 3 : 2 :: 17.481 : 11.654 = shorter diameter.

QUESTION 6.

Find the area of the triangle whose sides are 9, 8, and 6; thus, $\frac{9+8+6}{2} = 11.5$, and $\sqrt{11.5 \times 2.5 \times 3.5 \times 5.5} = \sqrt{553.4375} = 23.525$ square perches. Also, 6 A. 1 R. 12 P. = 1012 P., and 23.525 : 1012 :: 8^2 : 2753.1562 = square of the second side; therefore $\sqrt{2753.1562}$ = 52.47. Also,

8 : 9 :: 52.47 : 59.029 = longest side.

8 : 6 :: 52.47 : 39.353 = shortest side.

QUESTION 7. (Pl. 4, fig. 2.)

To find ABC:

AB	27.35	
BC	31.15	
CA	38.00	
	2)96.50	
Half sum	48.25	1.683497
Remainders	20.90	1.320146
	17.10	1.232996
	10.25	1.010724
		2)5.247363
ABC 420.417		2.623681

To find ACE:

AC	38.	
CE	40.10	
EA	22.20	
	2)100.30	
Half sum	50.15	1.700271
Remainders	12.15	1.084576
	10.05	1.002166
	27.95	1.446382
		2)5.233395
ACE 413.71		2.616697

To find CED:

CE	40.10	
CD	23.70	
DE	29.25	
	2)93.05	
Half sum	46.525	1.667686
Remainders	6.425	0.807873
	22.825	1.358410
	17.275	1.237408
		2)5.071377
CED 343.311		2.535688

Hence the whole area = 420.418+413.71+343.308 = 1177.436 Ch. = 117 A. 2 R. 38.976 P.

QUESTION 8.

Construction.

Make AB (Pl. 4, fig. 3.) equal to half the given perimeter = 52, and bisect it in D; make DC perpendicular to AB and equal to the square root of the given area; with the centre C and radius equal to AD, describe an arc cutting AB in E, complete the rectangle AEFG and it will be the one required. The demonstration is evident from Geometry.

Calculation.

$DE = \sqrt{CE^2 - CD^2} = \sqrt{676 - 480} = \sqrt{196} = 14.$
$AE = AD + DE = 26 + 14 = 40$, and $EF = EB = 26 - 14 = 12$.

QUESTION 9.

Construction.

Draw any line AC, (Pl. 4, fig. 4.) and in it take AE = 20 = given difference; make EF perpendicular to AC = 20; join AF and produce it to B, making FB = 20; then will AB be a side of the square.

Demonstration.

Since EA = EF, the angles FAE and AFE are each equal to half a right angle, and AC must be the diagonal of the square. Again the triangles CEF and CBF are equal, since they are right angled at E and B, and have the hypothenuse and one leg in each equal: we have therefore CE = CB = CA—20.

Calculation.

$AF = \sqrt{AE^2 + EF^2} = \sqrt{800} = 28.284$, and AB = AF+FB = 48.284; hence the area = AB^2 = 2331.344656 sq. per. = 14 A. 2 R. 11.34 P.

QUESTION 10.

Construction.

Let ABCD (Pl. 4, fig. 5.) be the given rectangle. In BA and BA produced take BH = BC, and AR = $\frac{3}{4}$ AD. On BR describe the semicircle BPR, meeting DA produced in P; bisect AH in O, and with the centre O and radius OP, describe the semicircle EPQ, make AG = AQ, complete the rectangle AF, and the thing is done.

Demonstration.

$AF = AE \times AG = AE \times AQ = AP^2 = AB \times AR = \frac{3}{4} AB \times AD = \frac{3}{4} AC$. Also, $BE = BH - HE = BC - AQ = AD - AG = GD$.

Calculation.

$AO = \frac{1}{2} AH = 10$: $AP^2 = AB \times \frac{3}{4} AD = 6000$; therefore, $OP = \sqrt{AP^2 + OA^2} = \sqrt{6100} = 78.1025$; $BE = BO - OE = 90 - 78.1025 = 11.8975$.

QUESTION 11.

Construction.

Let ABD (Pl. 4, fig. 6.) be the given circle. Draw the diameter AB and radius CD perpendicular to it; take $CF = \frac{4}{5} AC$; upon BF describe a semicircle cutting CD in E: with C as a centre and radius CE, describe the circle EGH, and the thing is done.

Demonstration.

CE is a mean proportional between CF and CB; hence $CF : CB :: CE^2 : CB^2 :: 4 : 5$; and since circles are as the squares of their radii, we have $GEH = \frac{4}{5} ABD$.

Calculation.

$$\sqrt{5} : \sqrt{4} :: AC\ (75) : EC = \frac{75\sqrt{4}}{\sqrt{5}}$$

$$= \frac{150\sqrt{5}}{5} = 30\sqrt{5} = 67.082, \text{ and } DE = DC - EC = 7.918.$$

QUESTION 12.

Construction.

With the given distances form the triangle ABC, (Pl. 4, fig. 7.) Upon AB describe the equilateral triangle ABD; join CD and on it describe the equilateral triangle CDE, which will be the one required.

Demonstration.

Since BD and BC are by construction two of the given distances; it is only necessary to prove that $BE = AC$, which is evident from the equality of the triangles DAC and DBE.

Calculation.

In the triangle ABC, find the angle BAC, thus,

BC	10		
AC	7.5	Ar. Co.	9.124939
AB	12.5	Ar. Co.	8.903090
	2)30.		
	15		1.176091
	5		0.698970
			2)19.903090
Cos. $\frac{1}{2}$ BAC	26° 34′		9.951545
BAC	53° 8′		

Then in the triangle DAC we have DA and AC, and the angle DAC = 113° 8′ to find DC, thus,

As DA+AC	20	Ar. Co.	8.698970
Is to DA—AC	5		0.698970
So is tang. $\frac{DCA+ADC}{2}$	33° 26′		9.819684
To tang. $\frac{DCA-ADC}{2}$	9° 22′		9.217624
ACD	42° 48′		

And,

As sin. ACD	42° 48′	Ar. Co.	0.167848
Is to sin. DAC	113° 8′		9.963596
So is AD	12.5		1.096910
To DC	16.92		1.228354

Then in CDE, we have the sides and angles to find the area thus,

As radius		Ar. Co.	0.000000
Is to sin. CDE	60°		9.937531
So is CD×DE	CD		1.228354
	DE		1.228354
To 2 CDE	247.88		2.394239

123.94 Ch. = 12 A. 1 R. 23.04 P.

Question 13.

Construction.

With the given bearings and distances protract the figure ABCDfg Pl. 4, fig. 8. Join Ag, and with the centres g and A, and distances equal to the 4th and 7th sides, describe arcs cutting in G. Join AG and gG, and draw DE, EF, and FG respectively parallel and equal to gG, Df, and fg. Then will ABCDEFG be the required map.

Calculation.

To find the bearing and distance of gA.

	Bearing.	Dist.	N.	S.	E.	W.
AB	S. 72 W.	24.00		7.42		22.83
BC	North.	38.00	38.00			
CD	N. 82½ E.	41.00	5.35		40.65	
Df	S. 80 E.	11.50		2.00	11.32	
fg	S. 26 W.	22.00		19.77		9.64
gA				(14.16)		(19.50)
			43.35	43.35	51.97	51.97

As diff. lat. 14.16 - - - - - - - - - Ar. Co. 8.848937
Is to departure 19.50 - - - - - - - - - - - - 1.290035
So is radius - - - - - - - - - - - - - - - 10.000000

To tang. bearing gA S. 54° 1′ W. - - - - - - 10.138972

As cos. bearing 54° 1′ - - - - - - - Ar. Co. 0.230955
Is to radius - - - - - - - - - - - - - - - 10.000000
So is diff. lat. 14.16 - - - - - - - - - - - 1.151063

To distance gA 24.10 - - - - - - - - - - - 1.382018

In the triangle AGg we have the sides to find the angles AgG and GAg;

Thus,

AG	37		
gG	20	Ar. Co.	8.698970
Ag	24.1	Ar. Co.	8.617982
	2)81.1		
Half sum	40.55	- - - - -	1.607991
Remainder	3.55	- - - - -	0.550228
			2)19.475171
Cos. $\frac{1}{2}$ AgG	56° 52½′	- - - -	9.737585
AgG	113° 45′		

And,

As AG 37 - - - - - - - - - -	Ar. Co.	8.431798
Is to gG 20 - - - - - - - - - - - -		1.301030
So is sin. AgG 113° 45′ - - - - - - - -		9.961569
To sin. gAG 29° 39′ - - - - - - - - - -		9.694397

Applying now the bearing of gA to these angles we will have the bearing of gG or DE = S. 59° 44′ E, and of GA = S. 83° 40′ W The area will then be calculated as in the following table, viz.

Sta.	Bearing.	Dist.	N.	S.	E.	W.	Cor. S.	Cor. W.	N.	S.	E.	W.	D. M. D.	N. Areas.	S. Areas.
AB	S. 72 W.	24.00		7.42		22.83	1			7.43		22.83	22.83		169.6267
BC	North.	38.00	38.00				1		37.99				00.00		
CD	N. 82½ E.	41.00	5.35		40.65		1		5.34		40.65		40.65	217.0710	
DE	S. 59¾ E.	20.00		10.08	17.28					10.08	17.28		98.58		993.6864
EF	S. 80 E.	11.50		2.00	11.32					2.00	11.32		127.18		254.3600
FG	S. 26 W.	22.00		19.77		9.64	1			19.78		9.64	128.86		2548.8508
GA	S. 83¾ W.	37.00		4.03		36.78	1			4.04		36.78	82.44		333.0576
			43.35	43.30	69.25	69.25									4299.5815 217.0710
															2)4082.5105
															40)2041.2553
													Area, 12 A. 3 R. 1.25 P.		4)511.25 P.
															12.3 R.

QUESTION 14.

Construction.

Make AB, (Pl. 4, fig. 9.) = the given side, and divide it in D, so that AD may be to DB in the ratio of 3 to 2; in AB produced, take DO a fourth proportional to AD—DB, DB, and AD, and with the centre O and radius OD, describe the semicircle DCE; make AG perpendicular to AB, and equal to twice the area divided by AB = 6; through G draw GF parallel to AB, cutting the circle in C and F; join AC BC, AF and BF; then will ABC and ABF answer the conditions of the question.

Demonstration.

Since AD—DB : DB : : AD : DO, we have AD : DB : : AO : DO or AO : AD : : DO : DB, therefore, AO : DO : : DO : OB, consequently (Euclid, F. 6.) AC : BC : : AD : DB : : 3 : 2; and AF : BF : AD : DB : : 3 : 2.

Calculation.

As 3+2 : 15 : : 3 : AD = 9, and DB = 6; also, 9—6 : 6 : : 9 : DO = 18, and AO = 9+18 = 27, join OC, and OF, and let fall the perpendiculars CL and FP; then OL = $\sqrt{OC^2—CL^2}$ = $\sqrt{324—36}$ = $\sqrt{288}$ = 16.9706, and AL = AO—OL = 10.0294; hence AC = $\sqrt{AL^2+LC^2}$ = $\sqrt{136.58886436}$ = 11.6871; and as 3 : 2 : : 11.6871 : BC = 7.7914. Again AP = AO+OP = 43.9706, and AF = $\sqrt{AP^2+PF^2}$ = $\sqrt{1969.41366436}$ = 44.3781; and as 3 : 2 : : 44.3781 : BF = 29.5854

QUESTION 15.

Construction.

Make AB, (Pl. 4, fig. 10.) = the given side, and BL = the sum of the other sides; Bisect AB in D, and take DH a third proportional to 2 AB and BL; Draw HE perpendicular to BH and equal to $\frac{1600}{50}$ = 32. Through E draw EF parallel and equal to BL; join EA and produce it to G, making FG = AB; draw AC parallel to FG, and join BC; then ABC is the triangle required.

Demonstration.

By Construction BL^2 = 2 AB × DH; also, in the similar triangles EGF and EAC, we have GF (AB) : AC : : EF (BL) : EC (HP).

Hence $BL \times AC = GF \times HP$, or $2\,BL \times AC = 2\,GF \times HP$. Subtracting these equals from the preceding, we have $BL^2 - 2\,BL \times AC = 2\,AB \times DH - 2\,GF \times HP = 2\,AB \times DP = (BP + AP) \times (BP - AP) = BP^2 - AP^2 = BC^2 - AC^2$. Hence $BL^2 - 2\,BL \times AC + AC^2 = BC^2$, and $BL - AC = BC$, or $BL = BC + AC$.

Calculation.

As 2 AB (100) : BL (85) :: BL (85) : DH = 72.25, and AH = DH — AD = 47.25. Now in the right angled triangle AHE, we have the sides AH and HE, to find HAE and AE; thus,

As AH 47.25	Ar. Co.	8.325598
Is to HE 32		1.505150
So is radius		10.000000
To tang. HAE 34° 6½′		9.830748

And,

As cos. HAE 34° 6½′	Ar. Co.	0.081981
Is to radius		10.000000
So is AH		1.674402
To AE 57.07		1.756383

Now in the triangle GEF we have FE, FG, and the angle FEG = HAE, to find FGE; thus,

As FG 50	Ar. Co.	8.301030
Is to FE 85		1.929419
So is sin. GEF 34° 6½′		9.748776
To sin. FGE 72° 25′		9.979225

Finally, in ACE we have AE and the angles to find AC; thus,

As sin. ACE 73° 28½′	Ar. Co.	0.018319
Is to sin. AEC 34° 6½′		9.748776
So is AE		1.756383
To AC 33.3793		1.523478

And BC = 85 — 33.3793 = 51.6207.

QUESTION 16.

Construction.

Make AC (Pl. 4, fig. 11.) = 50 = the given diagonal, and on it describe a semicircle ABC; make AE perpendicular to AC and $= \frac{1200}{50} =$ 24; draw EB parallel to AC, cutting the semicircle in B; join AB, BC, and draw CD and DA parallel to them; then will ABCD be the rectangle required.

Demonstration.

Since ABC is an angle in a semicircle, it is right, and ABCD is a rectangle. Also its area = AC × BF = 1200 perches = $7\frac{1}{2}$ acres.

Calculation.

FG = $\sqrt{BG^2 - BF^2} = \sqrt{49} = 7$; AF = AG—GF = 18, and AB = $\sqrt{AF^2 + FB^2} = \sqrt{900} = 30$, BC = $\sqrt{AC^2 - AB^2} = \sqrt{1600} = 40$.

QUESTION 17.

Construction.

Make AB (Pl. 4, fig. 12.) = the square root of the given area, and draw CE perpendicular to it; draw BC, making ABC = 30°, make AE = AC; bisect AC in D, and draw EF perpendicular to CE and = ED. Complete the parallelogram CEFG, which will be the one required.

Demonstration.

Since the angle B = 30°, and A = 90°, BC = 2 AC = CE = 4 CD, and EF = ED = 3 CD; therefore FC = $\sqrt{EF^2 + EC^2} = 5$ CD. Also $AB^2 = BC^2 - AC^2 = \frac{3}{4} BC^2 = \frac{3}{4} EC^2 = EC \times ED = EC \times EF =$ CEFG.

Calculation.

Since $AB^2 = \frac{3}{4} CE^2$ $CE^2 = \frac{4}{3} AB^2 = \frac{1}{3}$ the given area = 784, and CE = 28; hence EF = $\frac{3}{4}$ EC = 21.

QUESTION 18.

Construction.

With the given bearings and distances protract the figure ABCD, (Pl. 4, fig. 13.) and from B draw BP according to the given bearing

and distance of the spring. Produce DA and CB to meet in F, and through P draw EH parallel to AD. Bisect AF in G, join EG, and draw BM parallel to it, and MN parallel to FE. Make MT perpendicular to MN, and equal to the square root of the given area. Take MU a third proportional to MN and MT; draw UH parallel to MN, cutting AF in I; draw IK perpendicular to AF and equal to EP, and with the centre K and distance PH describe an arc cutting AD in Q; draw QPR, and the thing is done.

Demonstration.

In the similar triangles FGE and FMB, we have FB : FM :: FE : FG; therefore, 15.6, the triangle EFM = BFG; but EFM = ½ FMNE, and BFG = ½ BFA; hence FMNE = BFA. Again, because the triangles EPR, IQS, and PHS are similar, and the homologous sides EP (IK), IQ, and PH (KQ) form a right angled triangle, we have from Geometry EPR + IQS = SPH. Add FISPE to each, and we have FQR = EFIH. But FBA = EFMN, hence BAQR = MNIH = MN.MU = MT^2 = the given area.

Calculation.

From the bearings of the lines the angles may be found as follow. AFB = BEP = 23°, ABF = 84° 30′, BAF = 72° 30′, EBP = 145° 30′, and EPB = 11° 30′. Then, in the triangle EBP we have all the angles and side BP, to find EP and EB:

Thus,

As sin. BEP	23°	Ar. Co. 0.408122
Is to sin. EBP	145° 30′	9.753128
So is BP	7.90	0.897627
To EP	11.452	1.058877

And,

As sin. BEP		Ar. Co. 0.408122
Is to sin. BPE	11° 30′	9.299655
So is BP		0.897627
To BE	4.031	0.605404

Also, in the triangle ABF, all the angles and side AB are given, to find BF and AF;

Thus,

As sin. AFB 23°	Ar. Co.	0.408122
Is to sin. BAF 72° 30′		9.979420
So is AB 15.20		1.181844
To BF 37.101		1.569386

And,

As sin. AFB	Ar. Co.	0.408122
Is to sin. ABF 84° 30′		9.997996
So is AB		1.181844
To AF 38.722		1.587962

And FE = FB—BE = 33.07, and FG = ½ AF = 19.361;

Also,

As EF 33.07	Ar. Co.	8.480566
Is to GF 19.361		1.286927
So is BF 37.101		1.569386
To FM 21.721		1.336879

Now, in the parallelogram MIHN, we have MN = FE = 33.07, and IMN = F = 23°, and the area = 100 square chains, to find MI;

Thus,

As MN, sin. IMN	MN 33.07	Ar. Co.	8.480566
	sin. IMN 23°	Ar. Co.	0.408122
Is to radius			10.000000
So is MIHN			2.000000
To MI 7.739			0.888688

Therefore PH = EH — EP = FM + MI — EP = 18.008. Now, in the right angled triangle IKQ, we have IK = EP = 11.452, and KQ = PH = 18.008, to find IQ; thus,

KQ + KI = 29.46	log.	1.469233
KQ—KI = 6.556		0.816639
		2)2.285872
IQ = 13.898		1.142936

Hence AQ = FQ — FA = FM + MI + IQ — FA = 4.636.

QUESTION 19.

Construction.

With the given bearings and distances, protract the figure ABCD, (Pl. 4, fig. 14;) then, by Prob. 15, Chap. IV. divide ABCD into two equal parts by the line EF, parallel to CD; also, by the same problem, divide ABCD, and EBAF, each into two equal parts by the lines OM and PN, parallel to AD; join MN, produce it to I, and draw OH parallel to IM; join IH, then will EF and IH be the division lines required.

Demonstration.

Because PN is parallel to OM, we have IN : NM : : IP : PO : : IG : GH, because NG is parallel to HM; therefore, PG is parallel to OH, and consequently to IM. Now since OH is parallel to IM, we have IHM = IOM, to each add AIMD, and AIHD = AOMD = $\frac{1}{2}$ ABCD. In the same manner it may be shown that AIGF = $\frac{1}{2}$ ABEF = $\frac{1}{4}$ ABCD.

Calculation.

Draw EK and IL, each parallel to AD, and MU parallel to AB. From the given bearings find the angle A = 78° 30′, B = 139° 45′, C = 78° 45′, and D = 63°. By Prob. 15, Chap. IV., find EF and AF, thus,

As sin. C . sin. D	sin. C 78° 45′	Ar. Co.	0.008426
	sin. D 63° -	Ar. Co.	0.050119
Is to sin. A . sin. B	sin. A 78° 30′ - - - -		9.991193
	sin. B 139° 45′ - - -		9.810316
So is AB^2	AB 23 - - - - - -		1.361728
	AB - - - - - - - -		1.361728
To fourth term	383.274 - - - - - - -		2.583510
CD^2	2161.3201		
	2)2544.5941		
EF =	$\sqrt{1272.2970}$ = 35.67		

And in the triangle ECK,

As sin. E 38° 15′	Ar. Co.	0.208243
Is to sin. C 78° 45′		9.991574
So is CD—EF 10.82		1.034227
To FD 17.14		1.234044

AD 49.64

AF 32.50

Then in the triangle ECK, we have the angles and side EK = FD, to find EC, thus,

As sin. C 78° 45′	Ar. Co.	0.008426
Is to sin. K 63°		9.949881
So is EK 17.14		1.234044
To CE 15.57		1.192351

Consequently BE = BC—CE = 14.93. Now by the same problem find OM, AO, PN and AP, thus,

As sin. A . sin. D	sin. A	Ar. Co.	0.008807
	sin. D	Ar. Co.	0.050119
Is to sin. B . sin. C	sin. B		9.810316
	sin. C		9.991574
So is BC^2	BC 30.50		1.484300
	BC		1.484300
To fourth term	675.18		2.829416
AD^2	2464.1296		
	2)3139.3096		

$$OM = \sqrt{1569.6548} = 39.62$$

And,

As sin. RMD 38° 30′	Ar. Co.	0.205850
Is to sin. D		9.949881
So is AD—OM 10.02		1.000868
To AO 14.34		1.156599

And,

As sin. A . sin. F	sin. A - - -	Ar. Co.	0.008807
	sin. F - - -	Ar. Co.	0.050119
Is to sin. B . sin. E	sin. B - - - - -		9.810316
	sin. E - - - - -		9.991574
So is BE^2	BE 14.93 - - - -		1.174060
	BE - - - - - - -		1.174060
To Fourth term	161.78 - - - - -		2.208936

AF^2 - - - - 1056.25

2)1218.03

PN - - - $\sqrt{609.01} = 24.68$.

And,

As sin. RMD 38° 30′ - - - -	Ar. Co.	0.205850
Is to sin. F - - - - - - - - - - - -		9.949881
So is AF — PN 7.82 - - - - - - - - -		0.893207
To AP 11.19 - - - - - - - - - - -		1.048938

Hence OP = AO — AP = 3.15; wherefore we have

OM—PN (14.94) : PN (24.68) :: OP (3.15) : IP = 5.20; and AI = AP—IP = 5.99.

In the triangle MUL we have the angle U = A, L = D, and side MU = IO = IP + PO = 8.35, to find ML and UL; thus,

As sin. ULM 63° - - - - - - -	Ar. Co.	0.050119
Is to sin. MUL 78° 30′ - - - - - -		9.991193
So is MU 8.35 - - - - - - - - - -		0.921686
To ML 9.18 - - - - - - - - - - -		0.962998

And,

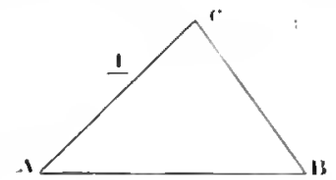
As sin. MLU 63° - - - - - - -	Ar. Co.	0.050119
Is to sin. UML 38° 30′ - - - - - - -		9.794150
So is MU 8.35 - - - - - - - - - -		0.921686
To UL 5.83 - - - - - - - - - - -		0.765955

Therefore IL = IU + UL = OM + UL = 45.45, and from the similar triangles ILM and OMH, we have

As IL (45.45) : LM (9.18) :: OM (39·62) : MH = 8.

Now, in the triangle ILH, we have the angle L = D, and sides IL and LH = LM + MH = 17.18, to find the angle LIH; thus,

As LI + LH	62.63	Ar. Co. 8.203218
Is to LI — LH	28.27	1.451326
So is tang. $\frac{\text{LHI} + \text{LIH}}{2}$	58° 30′	10.212681
To tang. $\frac{\text{LHI} - \text{LIH}}{2}$	36 23	9.867225
LIH	22 7	
Bearing IL	66 15	
" IK	S. 88 22 E.	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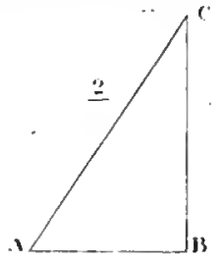
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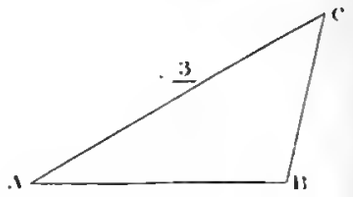
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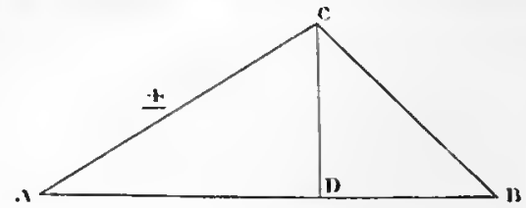
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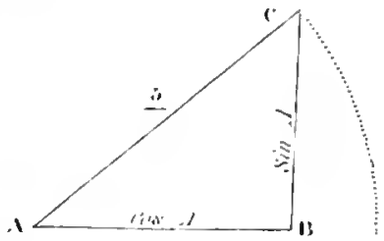
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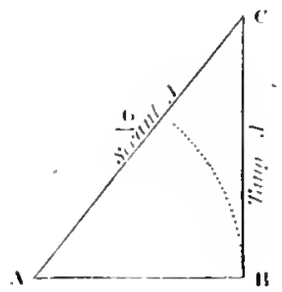
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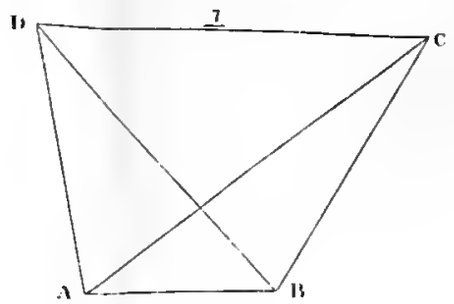
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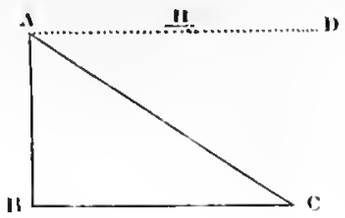
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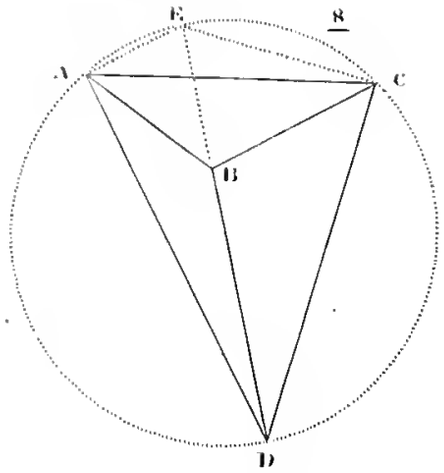
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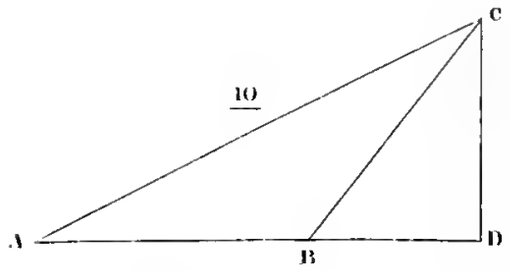
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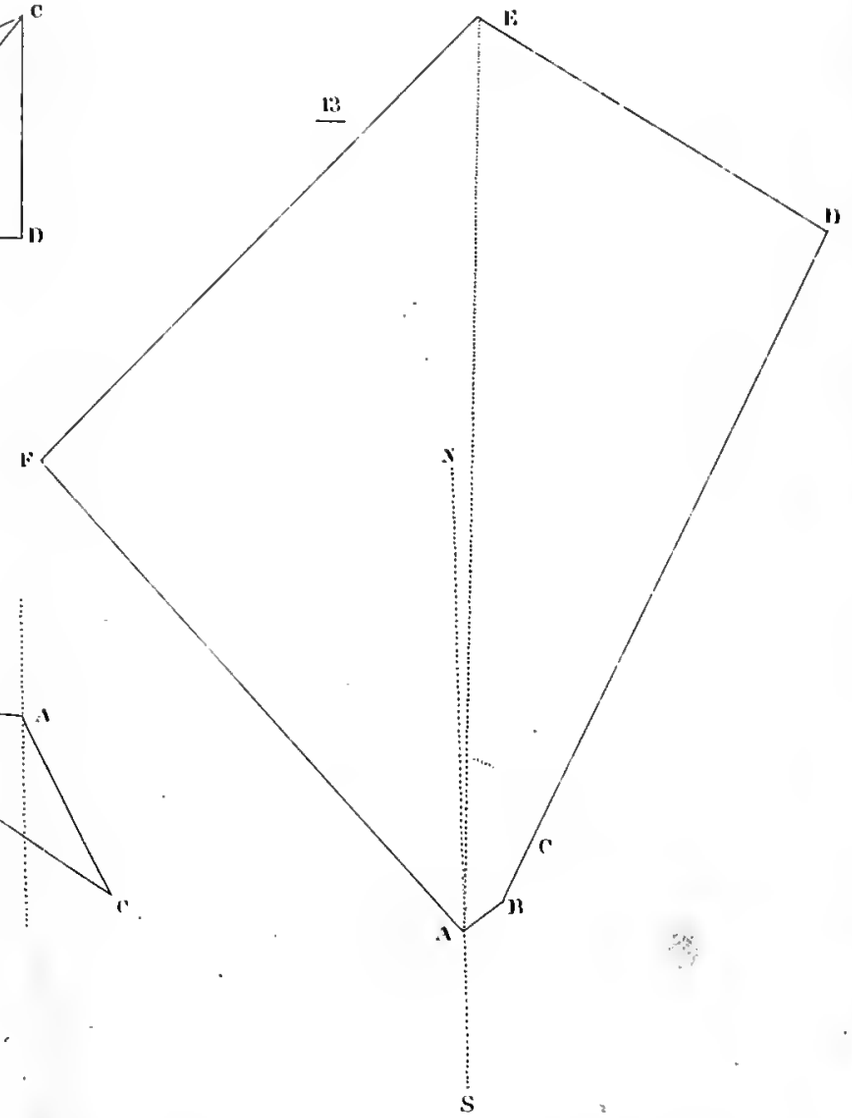
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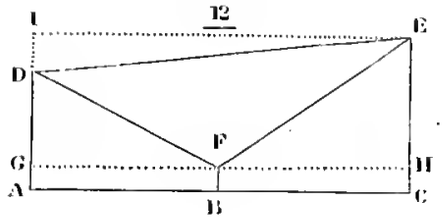
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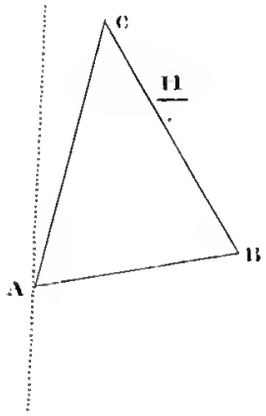
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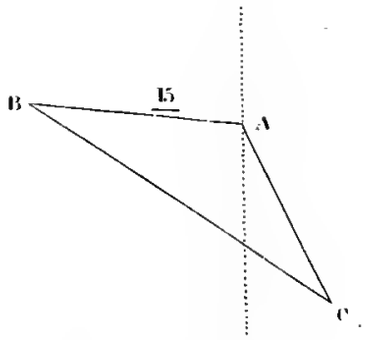
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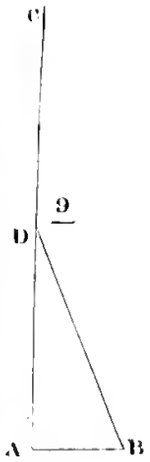
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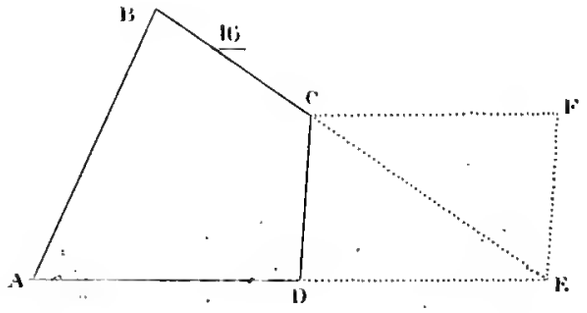
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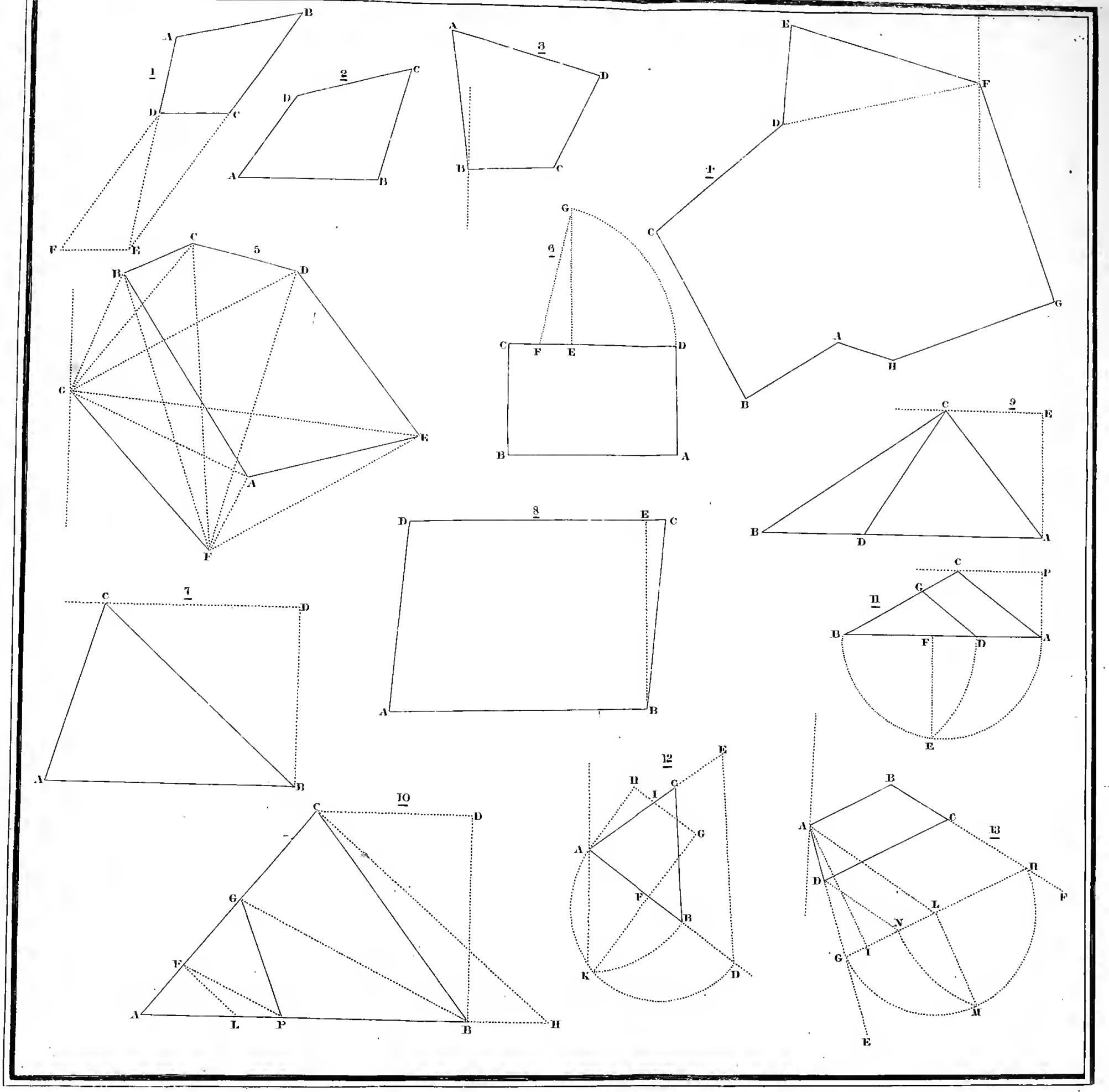
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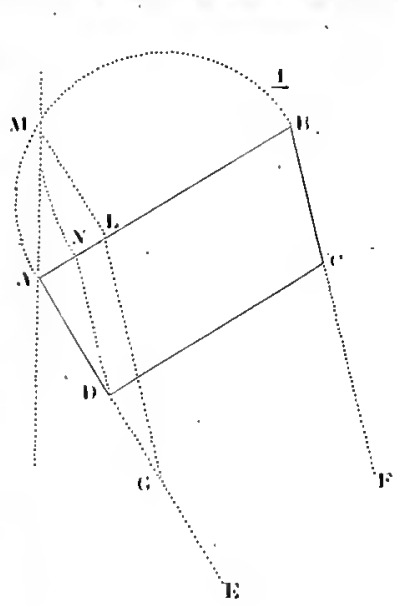
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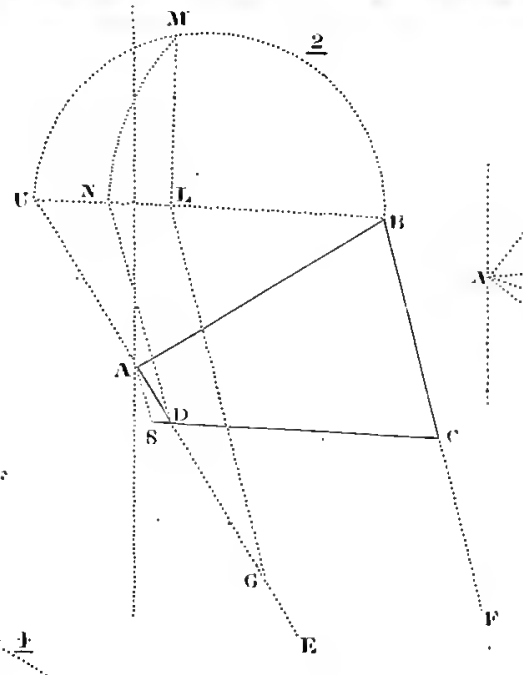
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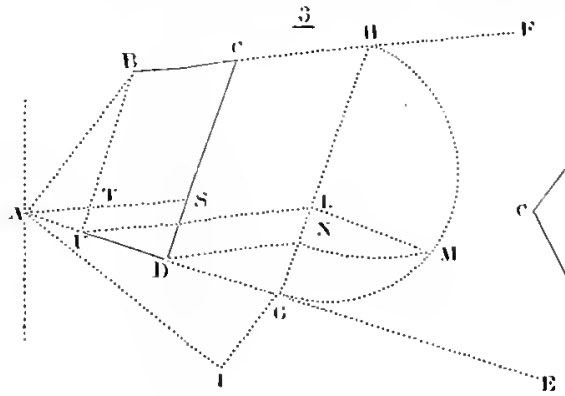
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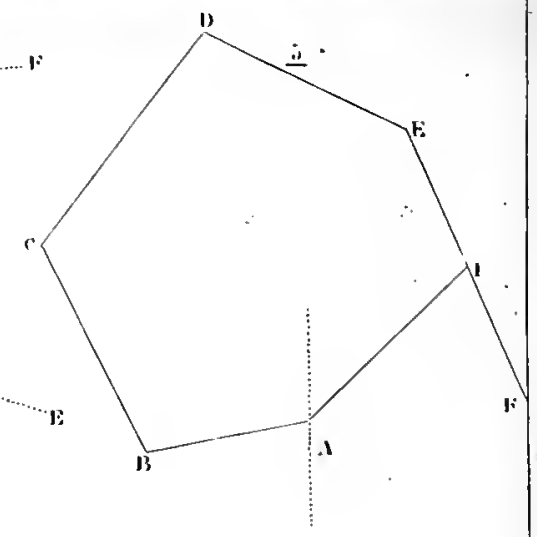
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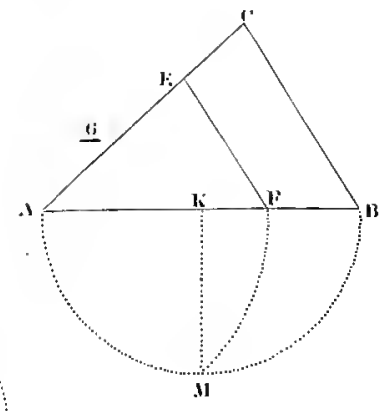
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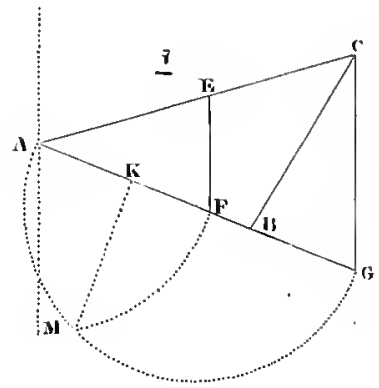
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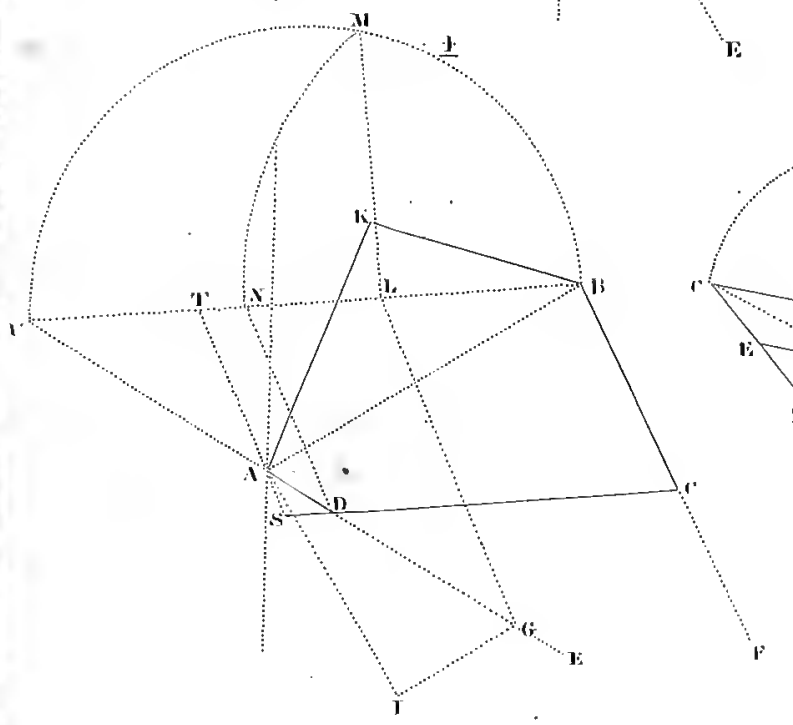
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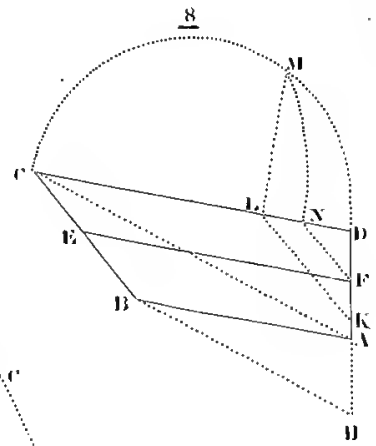
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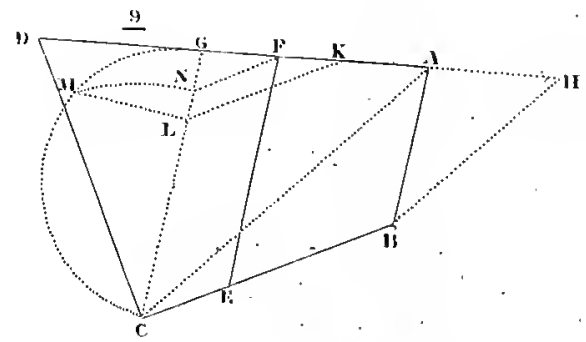
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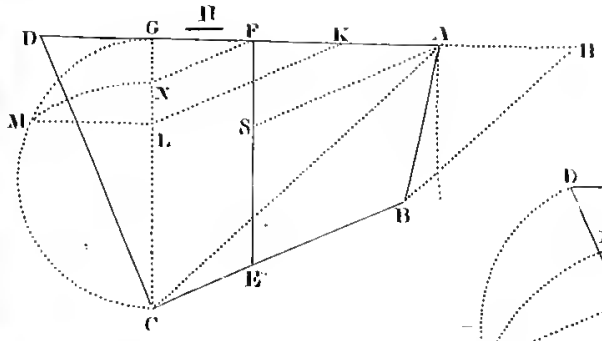
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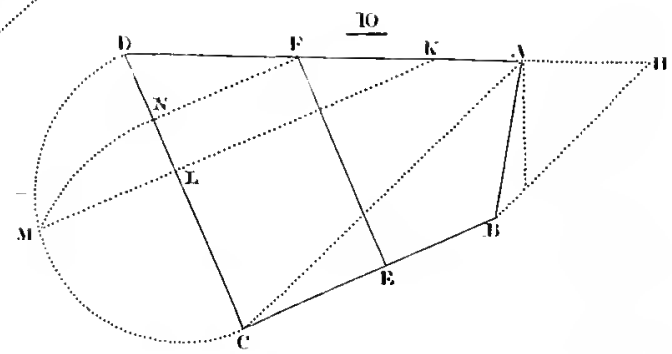
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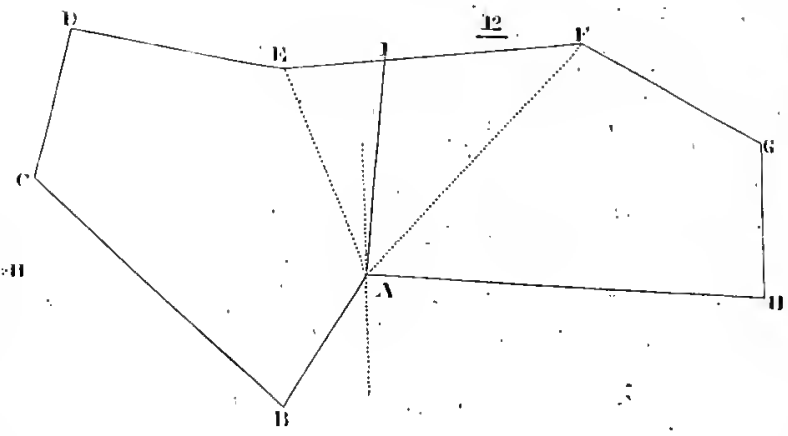
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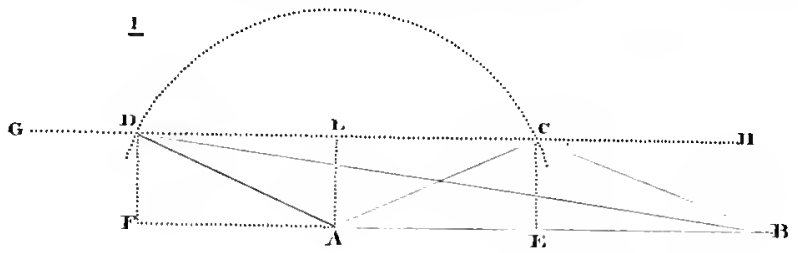

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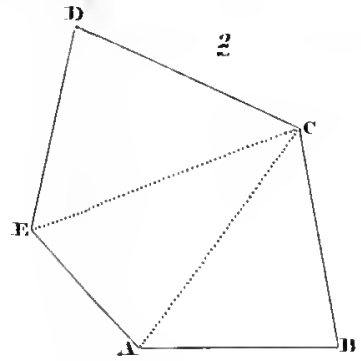

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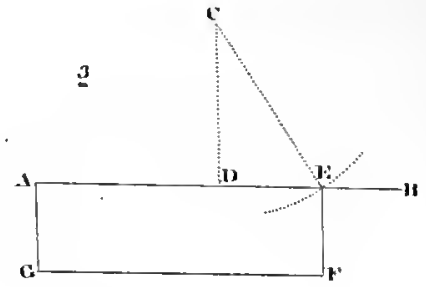

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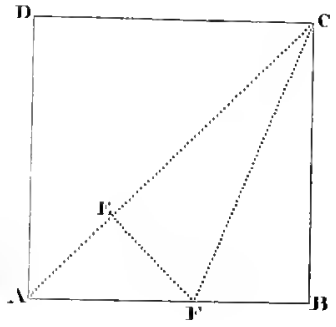

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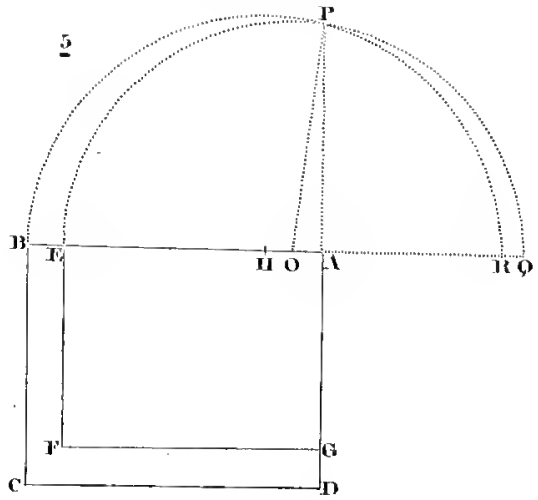

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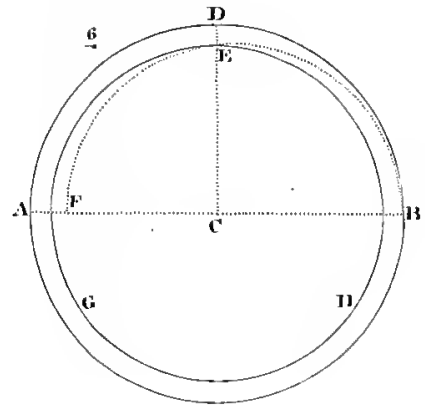

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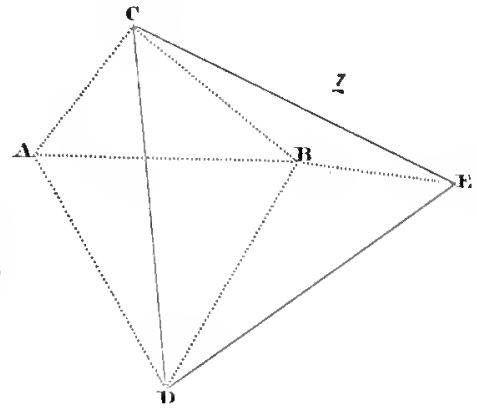

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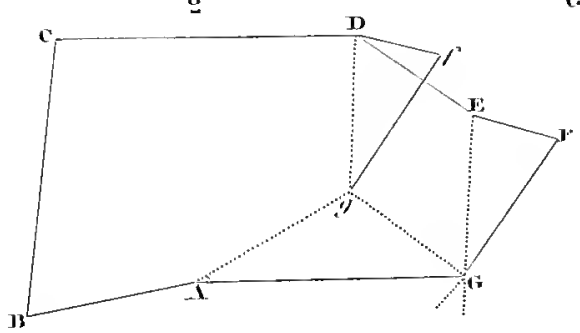

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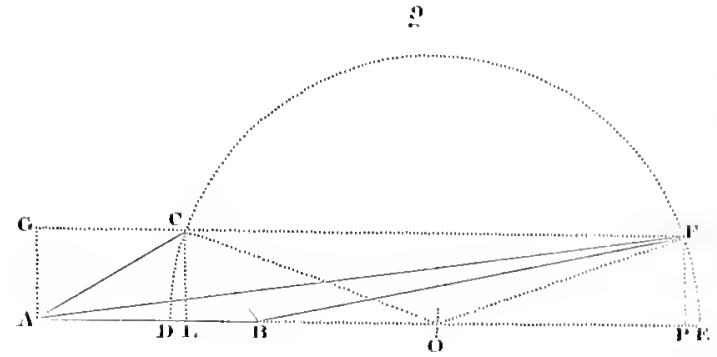

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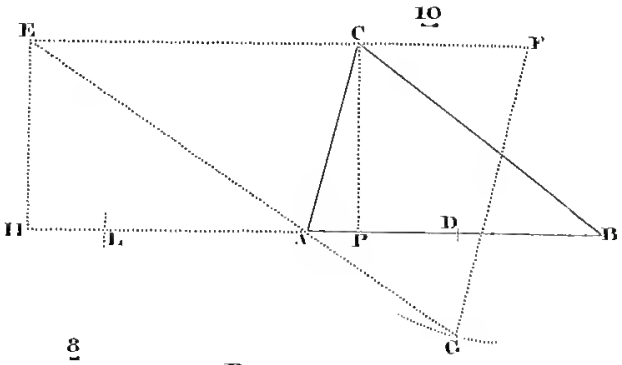

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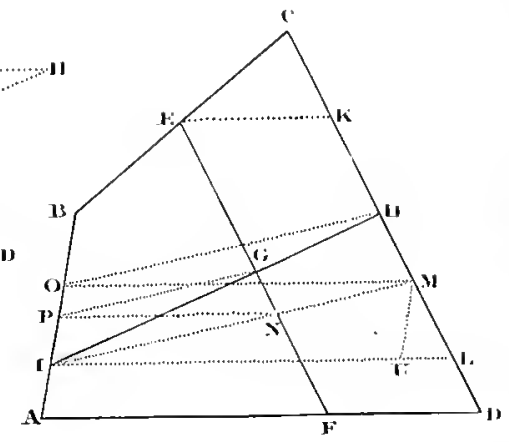

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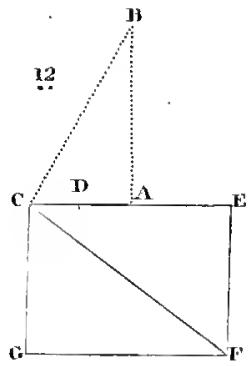

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